

Air Traffic

Team Problem

April 23, 2005

In this situation, consider the air traffic among these five airports in Washington: Bellingham, Seatac, Spokane, Walla Walla, and Yakima. The table gives the latitude and longitude for each airport. The second incomplete table shows in nautical miles the shortest distance between these airports.

Planes fly flight plans that take them on the shortest path to their destination, only varying from that path long enough to avoid a 2.5 nautical mile air space of another aircraft.

Assume that the earth has a radius of 3443.9 nautical miles. One minute of a degree of latitude is one nautical mile where as a degree of longitude varies in distance.

1. Graph the location of each airport. On the graph, use the first quadrant making the origin at $123^{\circ} 00' W$ and $46^{\circ} 00' N$, the positive x axis as east and positive y axis direction as North. Show both nautical miles and Lat/Long units on the axes.
2. Explain why the arc length of one degree of longitude varies and describe how to determine that relationship for various Lat/Long positions in the earth.
3. Determine the missing distances in the distance table.
4. If true North has bearing of 0° and true East is 90° , what is the bearing of the flight path from BLI to YKM?
5. An air transport company needs to pick up packages from each airport and distribute them to the other airports between the hours of 6 PM and 6 AM. The plane they intend to use flies at 150 knots (nautical miles per hour). It takes an average of 45 minutes on the ground at each airport to taxi in, dock, unload and load, taxi out, and take off. Determine whether the company can (and how best if it can) accomplish its task. Support your decision with mathematical analysis.
6. Assume air traffic is at the same altitude. During the daylight hours there are flights every hour (departing on the hour) from SEA to GEG and every hour (departing on the half hour) from GEG to SEA). These flights take 54 minutes of air time. How far from SEA do these planes pass each other?
7. Air traffic controllers have to be concerned when two planes are within 10 nautical miles of each other. Imagine a 10 nautical mile radius circle centered at the intersection of the SEA-GEG flight path and BLI-ALW flight path. What is the approximate probability that a random flight during daylight hours from ALW to BLI flying at 135 knots will be in this circle when one of the regularly scheduled flights between SEA and GEG in question 6 is also in this circle? Assume that there is an average of 12 hours of daylight per day.

Lat/Long Coordinates of Airports in Washington		
Airport	Latitude	Longitude
Bellingham (BLI)	$48^{\circ} 48' N$	$122^{\circ} 32' W$
Seatac (SEA)	$47^{\circ} 27' N$	$122^{\circ} 18' W$
Spokane (GEG)	$47^{\circ} 38' N$	$117^{\circ} 31' W$
Walla Walla (ALW)	$46^{\circ} 06' N$	$118^{\circ} 17' W$
Yakima (YKM)	$46^{\circ} 34' N$	$120^{\circ} 32' W$

Distance between Airports in Nautical Miles					
	BLI	SEA	GEG	ALW	YKM
BLI	0	82	213	237	
SEA	82	0	194	184	90
GEG	213	194	0	97	139
ALW	237	184	97	0	97
YKM		90	139	97	0

Scoring Rubric Summarized

Solve Problems using – Measurement – Probability – Number Sense

1. 5 points: Graphs and labels information
2. 3 points: Shows correct calculations/estimations of arc lengths
3. 2 points: Shows correct calculations of missing flight paths
4. 7 points: Shows correct calculations of bearing of flight path
5. 6 points: Determines most efficient flight plan and supports determination
6. 3 points: Determines location of passing flights
7. 4 points: Determines probability of flights being in the common airspace

Air Traffic Problem - Rubric for Scorers Team # _____ School _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated correctly (but wrong) points for the subsequent calculations should be awarded.

1. Graph (5 points)

- Communicates:**
- 1 pt – labels “Latitude” on vertical axis and “Longitude” on horizontal axis
 - 1 pt – scales are appropriate: horizontal 123° to 117° and 0 n.mi. to 210/250 n.mi.
vertical 46° to 49° and 0 n.mi. to 60 n.mi.
 - 1 pt – scales are consistent
 - 1 pt – labels latitude and longitude measurements as degrees and
labels distances as nautical miles
 - 1 pt – graphs data accurately

2. Arc length (3 points)

- Solves Problems:**
- Measurement 1 pt – explains why arc length for a degree longitude varies
 - 1 pt – determines arc length for one degree longitude at the equator is about 60 n.mi.
 - 1 pt - determines arc length for one degree longitude on the map is about 40 n.mi.

3. Determine distance (2 points)

- Solves Problems:**
- Measurement 1 pt – uses appropriate procedures
 - 1 pt – determines distance between BLI and YKM is about 160 n.mi.

4. Bearing of flight path (2 points)

- Solves Problems:**
- Measurement 1 pt – uses appropriate procedures
 - 1 pt – determines the bearing
from BLI to YKM is 31° east of south or true bearing of 149°

5. Pick-up and delivery (6 points)

- Reasoning and Solves Problems:**
- Number sense 1 pt – accounts for pick-up and delivery, i.e. two stops at each location except the first and last and the turnaround airport
 - Measurement 1 pt – accounts for 45 minutes at each stop
 - 1 pt – determines total time for route determined
 - 1 pt – determines a route that is possible within the allotted time
 - 1 pt – determines the shortest possible route.
 - 1 pt – shows another possible less desirable answer.

6. Meeting points (3 points)

- Solves Problems:**
- Measurement 1 pt – uses appropriate strategy and procedures
 - 2 pt – determines that planes meet at about 43 n mi and 151 n mi from SEA

7. Probability (4 points)

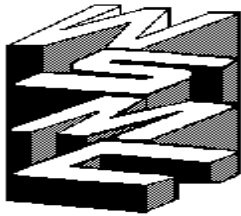
- Problem Solving:**
- Probability 1 pt – determines the amount of time a regularly scheduled flight SEA-GEG is in the circle during daylight hours (average 12 hours in a day)
 - 1 pt – determines the amount of time the random flight is in the circle
 - 1 pt – determines that the regularly scheduled flights don’t meet in the circle
 - 1 pt – determines the probability that a random flight and a regular flight would be in the circle

Overall (3 points)

- Communication:**
- 2 pt - presentation has a layout that is clear, organized, includes identifications, and sequencing is appropriate (1-partially meets given criteria, 0-meets none)
 - 1 pt - labels (degrees, nautical miles, etc.) are used sufficiently to demonstrate understanding and appropriate use

Total Points

28 points possible



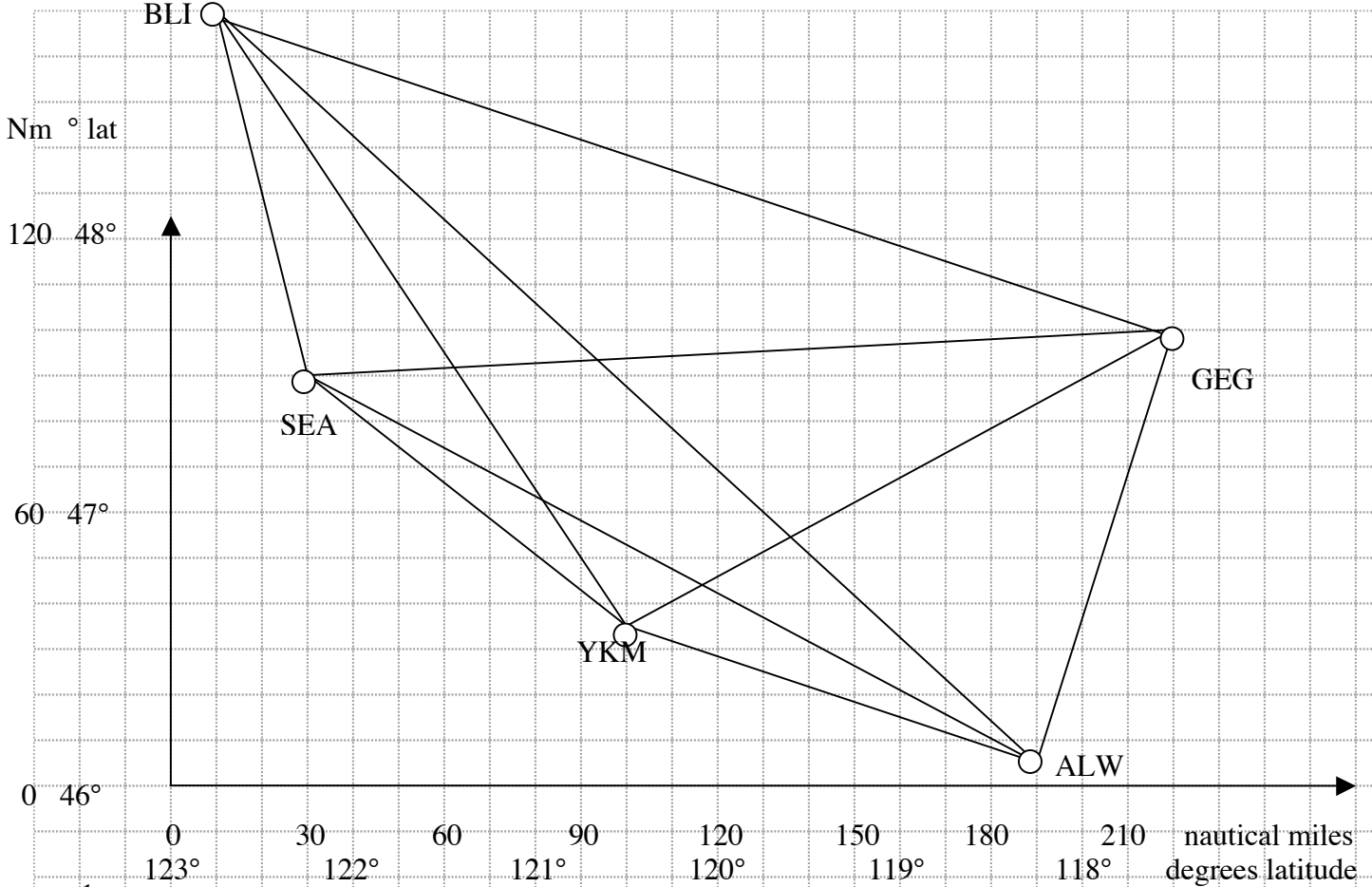
Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Name _____ **Sample Solution** _____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.



- 1.
2. Longitude lines are always great circles so they get closer together the farther a locations is from the equator. They are related to the radius of a circle at the latitude which is the cosine of the latitude. Therefore in Washington where latitudes are around 47° the $\cos(47^\circ)$ is 0.67 or 67% of the radius of the earth so one degree of longitude is 67% of 60 nautical miles or 40 nautical miles.
3. BLI and YKM differ 134' of latitude and 120' of longitude. Those distances in nautical miles are 134 and 80. Using the Pythagorean Theorem, the distance in nautical miles between BLI and YKM are 156.
4. Using the tangent inverse on the info from 3, the angle between south and the flight path from BLI and YKM is 31° east of south or true bearing of 149°.
5. The shortest loop to all airports is GEG-ALW-YKM-SEA-BLI-SEA-YKM-ALW-GEG which is 732 nautical miles. (This route could start in any of the five cities so isn't unique.) Flying at 150n mi/hr, this would take 4.88 hours. 45 minutes at the 8 airports is 6.00 hours totaling 10.88 hours, which is within the time frame stipulated. Another possible route might be BLI-SEA-YKM-ALW-GEG-BLI-SEA-YKM-ALW, 848 n mi for 5.633 hr plus 8 stops for 6 hours total 11.633 hrs but this pattern isn't repeatable each day with one plane so isn't as desirable.
6. $(194/54)t=194-194/54(t+30)$ gives $t=12$ min which is 43.1 miles from SEA. Also $(194/54)t=194-194/54(t-30)$ gives $t=42$ min which is 150.9 miles from SEA.
7. It takes a plane from SEA to GEG $20/(194/54)=5.6$ min to fly through 20 nm. Since this is not where SEA-GEG flights pass according to the graph, every hour one of two planes will be there for a total of ~132 min/12 hr. The ALW flight takes 8.9 min to cross 20 nm. Therefore $123/7200*8.9/7200=0.0002$ or 0.002% chance.