

WSMC High School State Competition

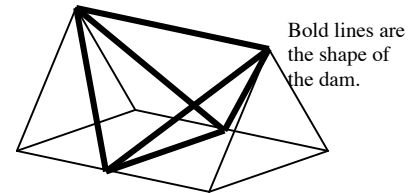
The Rattlesnake Dam Problem

Team Problem

March 16, 2005

Located in Eastern Washington, two dam sites could be considered for the Rattlesnake River. Both sites are indicated on the map. Before any decision is made to construct, a feasibility study is needed. Your job is to help with a portion of that study. In each case, briefly describe your method of estimation.

1. The Rattlesnake River *average* flow (volume per second) of water at any time during the year is periodic. The high flows *average* three times that of the low flows. During low flow, the river flows at 5 miles per hour and is approximately 20 feet wide and 2 feet deep. To maintain wildlife on the Rattlesnake, only 20% of the average flow is allowed to be stored in the reservoir for later diversion to agricultural use. Determine the amount of water that will be available for diversion to irrigation.
2. After the engineers determined that the large dam had a surface area of roughly 8.5 square miles and would hold approximately 2,000,000 acre feet of water at a cost of construction of approximately \$100 million, a citizens group did their own estimations and proposed the smaller dam. Estimate the surface area in acres of the smaller reservoir that would be a result of the smaller dam. The smaller dam would fill to the 1200 meter contour line.
3. Estimate the volume in acre feet of water that could be held in the smaller reservoir. (One acre foot of water is one foot of water covering one acre. There are 640 acres in a square mile and 0.30 meters in a foot.) Is the smaller reservoir large enough to hold the water to be diverted to irrigation?
4. The smaller dam was to be earthen with front and back slopes of 45 degrees. Estimate the cubic yards of fill required to make the dam. Assume the shape to be the bold lines in the figure.
5. The land that will be under water needs to be cleared before the reservoir is filled. Clearing five acres takes one crew one week to complete and costs \$15,000. The fill costs \$2 per cubic yard to be brought in, placed, and compacted. Determine the cost of building the smaller dam and clearing the reservoir.
6. It takes 2 acre feet of water to irrigate an acre of farmland per year. Using the volume of water available for irrigation (the 20%), estimate the cost per irrigable acre of land that would have to be charged to recover the costs of clearing the reservoir and building the smaller dam.



Scoring Rubric Summarized

Solve Problems using – Measurement – Statistics – Number Sense

✓ 7 points

Show how you arrived at the amount of water available for irrigation.

✓ 6 points

Show how you estimated the surface area and the volume of the small reservoir.

✓ 6 points

Show how you arrived at the estimation for the volume of the dam.

✓ 7 points

Show calculations for costs.

✓ 1 Reasoning

Communication

✓ 2 points

Present work in an organized, clear, and logical manner and label appropriately.

Rattlesnake Dam Problem - Rubric for Scorers Team # _____ School _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated appropriately (but of course the answer is incorrect) points for the calculations should be awarded.

1. Irrigation water (7 points)

	Solves Problems:	1 pt - calculate average low flow by multiplying cross-sectional area by the velocity ($20 \times 2 \times 5 = 200$ sq ft mi/hr)
	Measurement	1 pt - convert to cubic feet per second ($200 \times 5280 / 3600 = 293$ cfs)
	Statistics	1 pt - determine average flow is 10 mph or 2 times low flow (587 cfs)
	Number sense	1 pt - find 20% flow (117 cfs)
		1 pt - convert cubic feet to acre-feet (divide by 5280×5280 and multiply by 640)
		1 pt - convert "per second" to "per year" (multiply by $60 \times 60 \times 24 \times 365$)
		1 pt - determines that 84,700 acre feet of water will be available for irrigation

2. Area (3 points)

	Solves Problems:	1 pt - estimate area in square miles (3 to 4)
	Measurement	1 pt - describe or show estimation strategy
		1 pt - convert square miles to acres (~1800 to ~3000)

3. Volume of Reservoir (4 points)

	Solves Problems:	1 pt - estimate height in meters (100 to 150)
	Measurement	1 pt - convert meters to feet (330 to 500)
		1 pt - find volume in acre-feet (between 1/2 and 1/4 of the surface area times the depth, to get 148,000 to 750,000)
	Reasoning	1 pt - determines small reservoir has enough capacity

4. Volume of dam (6 points)

	Solves Problems:	1 pt - estimate distance across dam (2000' to 2500')
	Measurement	1 pt - determine volume of wedge in symbols or numbers ($1/2 Bh$)
		1 pt - determine volume of 2 end pyramids in symbols or numbers ($1/3 Bh$)
		1 pt - subtract volume of pyramids from volume of wedge
		1 pt - convert m to ft and ft to yd or cubic feet to cubic yd
		1 pt - determine volume of dam (2,700,000 to 7,700,000)

5. Cost to clear and fill (4 points)

	Solves Problems:	1 pt - divide the number of acres from #2 by 5 (360 to 600)
	Number sense	1 pt - multiply by \$15,000 (cost to clear: \$5.4 to \$9 million)
		1 pt - multiply the number of cu yd from #4 by \$2 (cost to fill: \$5.4 to 15.4 million)
		1 pt - add cost to clear and cost to fill ((\$10.8 to \$24.4 million)

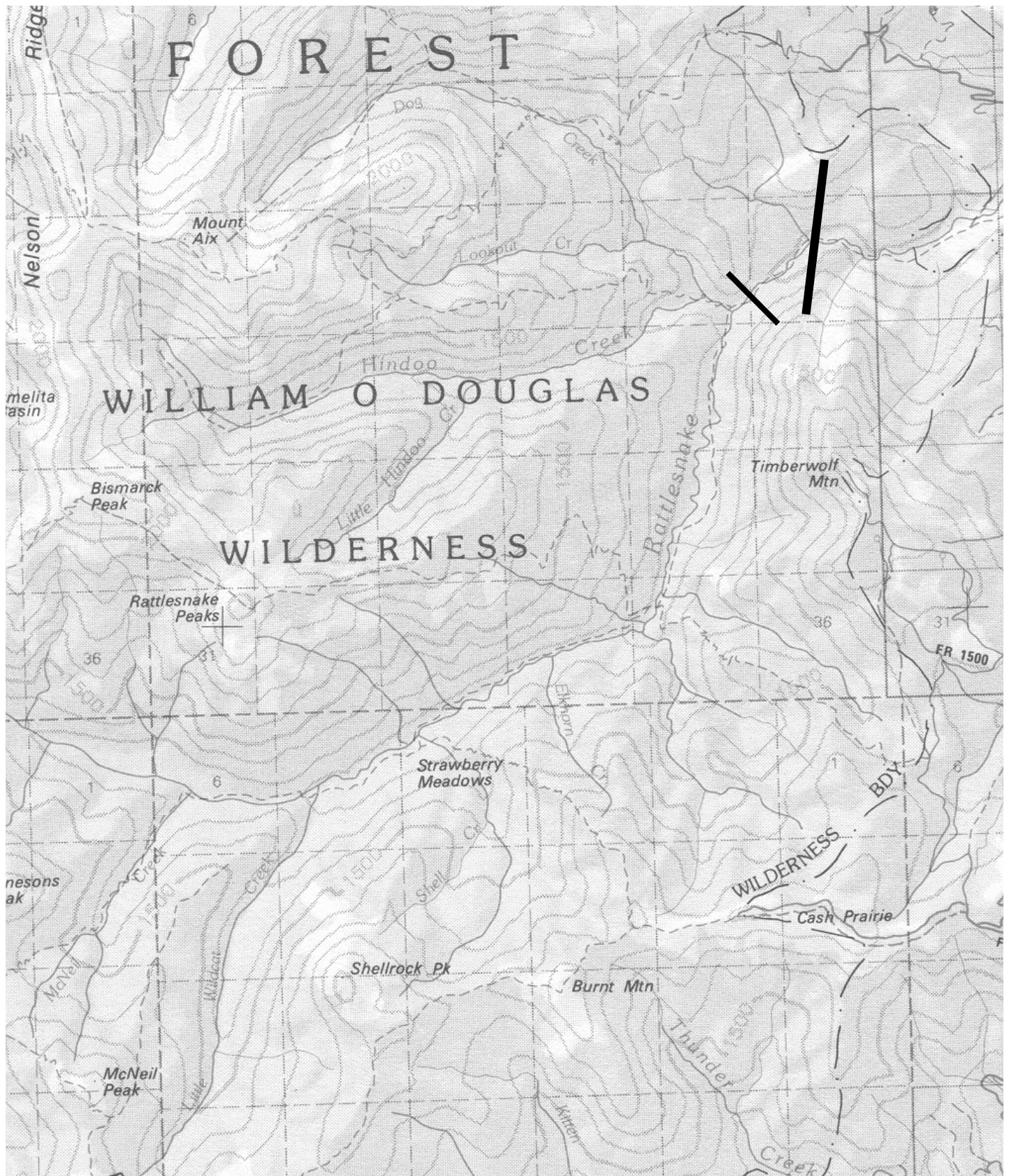
6. Cost per acre (3 points)

	Solves Problems:	1 pt - divide number of acre-feet from #3 by 2 (acres to irrigate: 42,000)
	Number sense	1 pt - divide the total cost from #5 by the number of acres to irrigate
		1 pt - determine the cost per acre (\$257 to \$581)

Overall (2 points)

	Communication:	1 pt - presentation has a layout that is clear, organized, includes identifications, and sequencing is appropriate
		1 pt - labels (m, ft, mph, acres, acre-ft, yd^3 , ft^3 , yd^3/sec , etc.) are used sufficiently to demonstrate understanding and appropriate use

	Total Points	29 points possible 20 points is the benchmark for automatic qualification for state for Division I schools and 18 for Division II schools.
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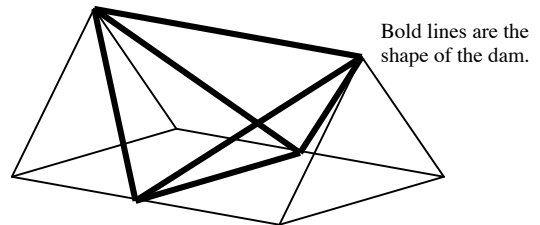


A square of the grid is approximately one square mile. The contour lines are in increments of 100 meters. This portion of 'Washington Atlas and Gazetteer,' copyrighted 2002, is copied with the permission of DeLorme Mapping Company, Freeport, MA.

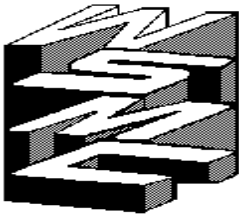
Rattlesnake Dam Problem Hints for the Junior Varsity Teams

1. Use the cross sectional area of the river and its flow rate to estimate the low flow rate. Then estimate the average annual flow of water, then the amount of water to be diverted.
2. After highlighting the 1200m contour line, estimate the square mile area of the smaller reservoir by use of the grid on the map (realizing that the actual area is a maximum at the surface and decreases from there.)
3. Estimate the depth of the smaller reservoir and some approximations to determine the approximate volume in acre feet of the smaller Reservoir. Then state whether the smaller reservoir is large enough to hold the water in part one.

4. Think of the dam as the bold portion of the shape. Calculate the volume of the entire figure and then subtract the volume of the two pyramids. Estimate the dimensions from the map.



5. From the previous calculations determine the costs.
6. Use your answers to prior questions to determine the number of acres that can be irrigated and the cost per acre.



Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

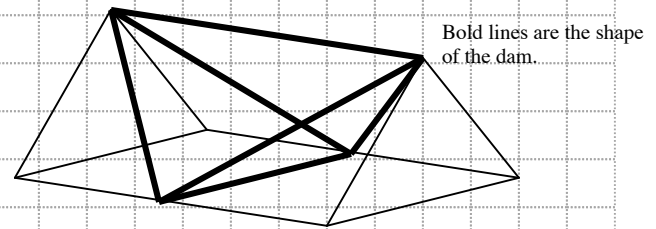
Name _____ **Sample Solution** _____

Name _____ Name _____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

1. The low flow is $20 \times 2.0 \times (5 \times 5280 / 3600) = 280$ cfs. The average flow is double this or 560 cfs. Twenty percent of this is 112 cfs which would be 81,000 acre feet per year.
2. By estimating the amount of each square mile that would be flooded to the 1200 m contour starting at the dam and working upstream, the fractions for the smaller reservoir are: $(0.7+0.7+0.6+0.4+0.3+0.1+0.2+0.3)$ sq mi = 3.3 sq mi or 2112 acres.
3. Estimating the smaller reservoir to have on average half that of the surface area and an average depth of 125 m, yields a volume of $1056 \times 125 / 3 = 440,000$ acre feet. The smaller holds 440,000 and a capacity only 81,000 acre feet is required. Therefore the smaller dam is sufficient to hold the irrigation water.

4. The dam is approximately the volume of the wedge less the two pyramids therefore its volume can be calculated by taking $\frac{1}{6}(\text{area of base} \times \text{height}) - 2 \times \frac{1}{3}(\text{area of the base} \times \text{height}) = \frac{1}{6}(\text{area of base} \times \text{height})$. The smaller is 2000 feet across and $125 / 3$ ft high so $\frac{1}{6}(2000 \times (2 \times 400) \times 400) / 27$ cu yds = 4,000,000 cu yds.



5. The costs for the smaller dam are \$6,000,000 for clearing (2112 acres/5 acres x \$15,000 per unit) and \$8,000,000 for fill (4,000,000 x \$2 /cu yd) yielding a total cost of approximately \$14,000,000.
6. Since only the smaller dam needs to be built and only approximately 40,000 acres can be irrigated, the cost per acre would be $\$14,000,000 / 40,000$ acres = \$350/acre.