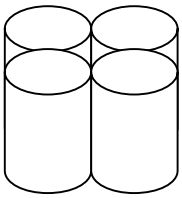
WSMC High School Regional Competition

Water Bottle Packaging

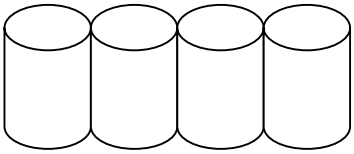
Team Problem

March 8, 2006

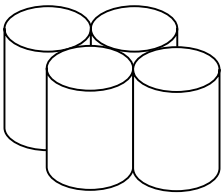
The Hydration Water Company is looking at ways to package their product, cylindrical bottles of water, in four packs. The Design Team has done some exploring of ideas and has given you the task of evaluating designs. The Design Team has proposed four different packaging ideas: four bottles in a square based box, four bottles is a rectangular based box, four bottles is a non-rectangular parallelogram based box, and four bottles in a circular based box.



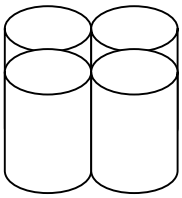
Square base



Rectangular base



Parallelogram base



Circular base

Each box has a bottom and top. Assume the bottles are cylindrical in shape with a 6 cm diameter. You are free to modify the height, '**h**', of the bottles.

- **A.** Determine the volume of each box as a function of height.
- **B.** Determine the volume efficiency ratio for each box, '**R**', defined as the ratio of the volume of the bottles to the volume of the box.
- **C.** Rank the packaging choices according to their efficiency ratios.
- **D.** Determine the surface area of each box as a function of height.
- **E.** Assume there is no waste in making the boxes and that the cost of the box material is '**C**' cents per square centimeter. Write the cost for each box and rank the packaging choices according to the costs.
- **F.** What other factors related to this problem might influence the packaging decisions?

Be sure to show all of your work. Organize and label the sections of your work including your data and diagrams.

Scoring Rubric Summarized

Solve Problems using – Measurement – Algebraic Sense – Number Sense – Geometric Sense

- ✓ 7 points - Show how you got the volume formulas.
- ✓ 7 points - Show how you got the efficiency ratios and ranked them.
- ✓ 5 points - Show how you got the surface area formulas.
- ✓ 5 points - Show box costs and how you compared and ranked them.

Makes Connections

- ✓ 2 point - Identify other factors that might influence the packaging decision.

Communication

- ✓ 3 points - Present work in an organized, clear, and logical manner, label appropriately, and use mathematical language and notation.

Water Bottle Problem - Rubric for Scorers Team # _____ School _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated appropriately (but of course the answer is incorrect) points for the calculations should be awarded.

A. Volumes as Functions of the Height (7 points)

Solves Problems:	1 pt – Volume of square based box
Measurement	1 pt – Volume rectangular based box
Geometric Sense	1 pt – Height of rhombus
	1 pt – Side of rhombus
	1 pt – Volume rhombus based box
	1 pt – Radius of circular based box
	1 pt – Volume of the circular based box

B. Efficiency Ratios (5 points)

Solves Problems:	1 pt – Determined the volume of the bottles accurately
Measurement	1 pt – Calculated the efficiency ration for the square based box
Number Sense	1 pt – Calculated the efficiency ration for the rectangular based box
	1 pt – Calculated the efficiency ration for the rhombus based box
	1 pt – Calculated the efficiency ration for the circular based box

C. Ranks According to the Ratios (2 points)

Solves Problems:	2 pt – Nearly correct on the rankings, 1pt – At least two are ranked correctly
Number Sense	

D. Surface Areas as Functions of the Height (5 points)

Solves Problems:	4 pt – In the process of calculating the surface areas, work shows evidence that the lateral area
Measurement	and the area of the base were calculated. Also, the lateral area and one or two bases
Algebraic Sense	were totaled, two bases being correct. (Award 1 or 0 points for **each** box)
	1 pt – Accounted for two bases when calculating the surface area

E. Cost Functions and Comparisons (5 points)

Solves Problems:	1 pt – Multiplied the surface areas found in part D by **'C'**.
Algebraic Sense	1 pt – Wrote the cost function in a form for ease of comparing
Number Sense	3 pt – Compared costs of boxes for specific domain values (1 point for each correct comparison based on their work, maximum of 3 points)

F. Describes which box is best under each condition (2 points)

Solves Problems	2 pt – Identifies significant factors important in the packaging decision such as efficiency of
Makes Connections	stacking the boxes on pallets, reasonable of heights of water bottles, etc. (Each significant factor earns a point, maximum of 2 points.)

Overall (3 points)

Communication:	1 pt – Is organized and easy to follow
	1 pt – Uses diagrams and/or tables to help
	1 pt – Uses appropriate mathematical language and/or notation

Total Points	29 points possible
	20 points is the benchmark for automatic qualification for state for Division I schools and 18 for Division II schools.

Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

Sample Solution	

A Volume	$12^2 h$	$24 \cdot 6h$	$(6+3\sqrt{3})(6+4\sqrt{3})h$	$(3+3\sqrt{2})^2 \pi h$
B Efficiency Ratio **R**	$\frac{4 \cdot 9\pi h}{12^2 h} = \frac{36\pi}{144} = \frac{\pi}{4} \approx 0.79$	$\frac{4 \cdot 9\pi h}{24 \cdot 6h} = \frac{36\pi}{144} = \frac{\pi}{4} \approx 0.79$	$\frac{4 \cdot 9\pi h}{(6+3\sqrt{3})(6+4\sqrt{3})h} = \frac{36\pi}{72+42\sqrt{3}} = \frac{6\pi}{12+7\sqrt{3}} \approx 0.78$	$\frac{4 \cdot 9\pi h}{(3+3\sqrt{2})^2 \pi h} = \frac{36}{27+18\sqrt{2}} = \frac{4}{3+2\sqrt{2}} \approx 0.686$
C Rank of Efficiency Ratios	Best Tie	Best Tie	Second best/worst	Worst
D Surface Areas	$2(144)+48h =$ $288+48h$	$2(144)+60h =$ $288+60h$	Using info above: $2(72+42\sqrt{3})+(24+16\sqrt{3})h \approx$ $289.5+51.7h$	$2\pi(27+18\sqrt{2})+(6+6\sqrt{2})\pi h \approx$ $329.59+45.5h$
E Costs	(288+48h)**C**	(288+60h)**C**	(289.5+51.7h)**C**	(329.59+45.5h)**C**
E and F Descriptions by one editor	This is the best except for boxes with heights greater than 16.7 cm. At that point the cylindrical box becomes less costly.	Except for heights less than 0.18 cm (which is unreasonable) when this box uses less material than the Rhombus based box, this is better then the cylindrical box until a height of 2.87 cm (again an unreasonable height).	For all practicalities this is better than the rectangular based box but worse than the square based box. It is better than the cylindrical box until the height reaches 6.5 cm which is still awfully short for a bottle of water. Doesn't stack well on a pallet.	This package starts out the worst but gets better as the height of the bottles increases, passing the rectangular based box at 0.18 cm, the rhombus based box at 6.5 cm, and the square based box at 16.7 cm. However this shape would not fit well on a shelf or pallet and end up costing more overall. Doesn't stack well on a pallet.
E and F Descriptions by another editor Notice how rounding changes results.	Used 288+48h Sq<Rect. $\forall$ h<21cm Because same intercept, smaller slope. Sq<Cyl if h<21cm Sq<Rh $\forall$ h because larger intercept larger slope for rhombus	Used 288+60h Rect<Sq Rect<Rh if h<.125cm from 288+60h<289+52h Rect<Cyl if h<3cm from 288+60h<330+46h	Used 289+52h Rect<Cyl if h<6.8 from 289+52h<330+42h Rh is never <Sq Rh<Rect if h>0.125 cm See previous work	Used 330+46h Cyl<Rect if h>3cm Cyl<Sq when h>21 Cyl<Rh if h>6.8cm See previous work
Other considerations	Sq<REct<Rh<Cyl for 0 to 0.125 cm …Height too short to be useful Sq<Rh<Rect<Cyl for 0.125 to 3 cm …Height too short to be useful Sq<Rh<Cyl<Rect for 3 to 6.8 cm … Height too short to be useful Sq<Cyl<Rh<Rect for 6>8 to 21 cm … This is a small bottle but could be used Cyl<Sq<Rh<Rect for more than 21 cm … Close to normal size found in markets. However Cylinders and Rhombus bases are not going to be as efficient when stacking them on Pallets for example. Recommendation would be bottles between 9 cm and 21 cm in the square base format.			

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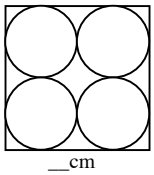
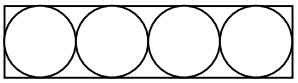
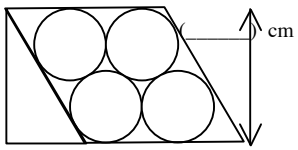
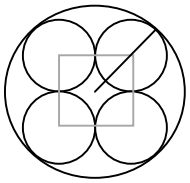
	 __cm  __cm __ cm  (_____) cm 30-60-90 Δ with hypotenuse of (_____) cm  Radius= (_____) cm V of cylinder is $\Pi r^2 h$ SA=2 Πr^2 +2πrh			
A Volumes				
B Efficiency Ratios				
C Rank of Efficiency Ratios				
D Surface Areas				
E Costs				
Which is better? (For what heights?)				
F Other consider-ations				

Special answer sheet for Junior varsity teams. Support all your work with clear and convincing information and calculations.