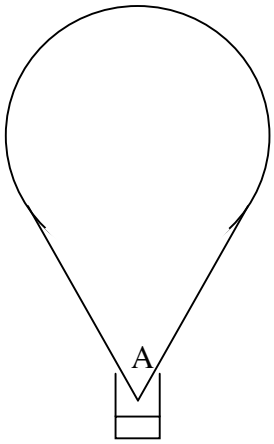
WSMC High School Regional Competition

Hot Air Balloon

Team Problem

March 7, 2007

Hot air balloons were given a new life with the advent of nylon and other technologies. A typical hot air balloon is roughly the shape of a sphere with a cone tangent to it as in the figure. To lift any weight, the balloon must be lighter than air it displaces. In fact the total weight of the balloon (including the trapped air) equals the weight of the air displaced by the balloon when the balloon is floating.



For the rest of this problem use this typical balloon: The nylon that makes up the balloon, called the envelope, weighs about 250 pounds. The burner that heats the air in the balloon weighs 50 pounds and the propane tanks weigh about 405 pounds. The basket is 140 pounds. A balloon capable of lifting a payload of 750 additional pounds typically has a volume of 100,000 ft^3.

Because the bottom of the balloon is open, the air pressure in the balloon is equal to the atmospheric pressure surrounding the balloon. At sea level atmospheric pressure is approximately 14.7 pounds per square inch. One cubic foot of air at 60ºF weighs about 28 grams. (One kilogram converts to 2.2 pounds on earth.) If the volume and pressure are constant, the Ideal Gas Law implies that the weight of air in the balloon is inversely proportional to the temperature of the air in the balloon if the temperature is measured using the Kelvin temperature scale. The relationship between the Kelvin temperature scale and the Fahrenheit temperature scale is linear.

A. What is the weight to volume ratio of air in pounds per cubic foot at sea level?
B. What is the weight in pounds of the air displaced by the balloon?
C. If the balloon is lifting just enough to lift the balloon and basket off the ground with a full payload, what is the weight of the air inside the balloon?
D. If water freezes at 32ºF and 273ºK and boils at 212ºF and 373ºK write a conversion formula for changing Kelvin temperature to Fahrenheit and another for Fahrenheit to Kelvin.
E. Write an equation relating the weight in pounds of the air in the balloon to the temperature of the air in the balloon in degrees Fahrenheit.
F. If the outside air temperature is 60ºF, what must be the temperature of the air in the balloon?
G. If the weight of the air decreases 1% for every 1000 feet of elevation and the maximum temperature the air in the balloon can be is 250ºF, could this balloon take a 150 pound load to the summit of Mt. Everest (29,000 feet)? Explain.
H. If angle A is 90º, find the distance from A to the top of the balloon in terms of the radius of the sphere. Then estimate the radius of the spherical portion of the balloon. Provide justification for your estimate including any assumptions you made.

Be sure to show all of your work. Organize and label the sections of your work including your data and diagrams.

Scoring Rubric Summarized

Solve Problems using – Measurement – Algebraic Sense – Number Sense – Geometric Sense

A. 2 points - Show how you got the weight/volume ratio.
B. 1 point - Show how you got the weight of displaced air.
C. 2 points - Show how you got the weight of captive air.
D. 4 points - Show how you got the temperature conversion formulas.
E. 4 points - Show how you got the weight vs. temp formula.
F. 3 points - Show how you determined air temp in balloon.
G. 3 points – Determine and explain height expectations.
H. 6 points – Show how you approximate the sphere radius
4 points - Present work in an organized, clear, and logical manner, label appropriately, and use mathematical language and notation.

Hot Air Balloon - Rubric for Scorers Team # _____ School _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated appropriately (but of course the answer is incorrect) points for the calculations should be awarded.

A. Weight to Volume Ratio of Air (2 points)

Measurement	1 pt – Uses appropriate procedure to convert grams to pounds 1 pt – Gives weight to volume ratio (0.0616 lb/ft^3)

B. Weight of the Displaced Air (1 points)

Solves Problems **Measurement**	1 pt – Determines weight of the air displaced by balloon (6160 lb)

C. Weight of the Captive Air (2 points)

Solves Problems	1 pt – Demonstrates need to remove weight of equipment and payload from weight of displaced air 1 pt – Correctly determines the weight of the captive air (4565 lb)

D. Temperature Conversion Formulas (4 points)

Solves Problems **Algebraic Sense**	2 pt – Determines the K to F formula (one point if 'slope' only is correct) F = (9/5)K - 459 2 pt – Determines the F to K formula (one point if 'slope' only is correct) K = (5/9)F + 255

E. Weight vs. Temperature Formula (4 points)

Solves Problems **Algebraic Sense** **Measurement**	1 pt – Demonstrates understanding of inverse model as either y=k/x or xy=k 1 pt – Determines constant of variation using appropriate data but may not be correct because of unit conversion (1,777,000) 1 pt – Writes an equation that relates weight to temperature 1 pt – The equation relates weight in <u>pounds</u> to temperature in <u>Kelvin</u> (W=1777000/K)

F. Air Temperature in Balloon (3 points)

Solves Problems **Algebraic Sense** **Makes Connections**	2 pt – Uses appropriate procedures and their answer(s) from part E above (award one point for partially correct work) 1 pt – Determines air temperature in the balloon (242^o)

G. Maximum Altitude (3 points)

Solves Problems **Algebraic Sense** **Reasoning**	1 pt – Determines the weight of air in balloon + equipment + payload (5505 lb) 1 pt – Determines the weight of the displaced air at 29,000 feet (4374 lb) OR maximum height of 11,000 ft 1 pt – Explains why the balloon with 150 lb payload cannot reach 29,000 ft

H. Dimensions of Balloon (6 points)

Solves Problems **Geometric Sense** **Algebraic Sense**	1 pt – Shows recognition that this balloon is a cone overlapping a sphere 1 pt – Determines the height of the cone is radius divided by the square root of two 1 pt – Determines the height of the balloon in terms of the radius ($r + r\sqrt{2}$) 1 pt – Demonstrates the volume of the balloon is the sum of a sphere and cone volumes less the overlap 1 pt – Uses a reasonable method to account for the volume of the overlap or the part of the cone outside the sphere 1 pt – Determines the radius of the balloon (28-29 ft approximately

Overall (4 points)

Communication	1 pt – Work is organized, labeled, and easy to follow 1 pt – Uses diagrams to help explain/clarify the work 1 pt – Uses appropriate mathematical language and/or notation 1 pt – Consistently uses units correctly

Total Points	29 points possible 20 points is the benchmark for automatic qualification for state for Division I schools and 18 for Division II schools.

Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

Sample Solution

A. The weight/volume is: (28g*0.001 kg/g*2.2 lbs/kg)/1 ft^3 = 0.0616 lbs/ ft^3.

B. The weight of 100,000 ft^3 of air at 60 degrees is 0.0616 lbs/ ft^3*100,000 ft^3=6,160 lbs.

C. The weight of the parts of the balloon, the payload, and the trapped air equals the weight of the displaced air so 6,160 lbs – (250+50+405+140+750) lbs = 4,565 lbs.

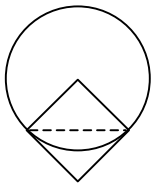
D. Using the two points (32, 273) and (212, 373) and finding the line through these points we get: F=1.8K-459 and K=(5/9)F+255.

E. Since they are inversely proportional, (the weight in pounds)*(the temperature in Kelvin) = a constant so: 6160[(5/9)60+255]=1777000 so W[(5/9)F+255]=1777000.

F. Since the weight of the air in the balloon is 4,565 lbs, part E yields 4,565[(5/9)F+255]=1777000. Solving for F we get about 242ºF.

G. At 250ºF, the weight of the balloon would be {1777000/ [(5/9)250+255]+ (250+50+405+140+150)} lbs = 5466 lbs. Solving $5466=6160(0.99)^x$ we get x to be about 12. Therefore the maximum height is about 12(1000 ft) = 12,000 ft. Or the weight of the displaced air at the summit of Mt. Everest would be $6160(0.99)^{29}$ = 4602 lbs so can't make it.

H. Using the diagram, we see that A to the top is $(1+\sqrt{2})$*radius. Using the cone with the dotted segment as the diameter of the base and the center of the sphere as the vertex, we get a volume of $(1/3)\pi(\text{radius}/\sqrt{2})^3$. This would equal the volume of the inverted cone. The volume of the balloon would be the volume of the cone plus the volume of the sphere less the curved portion of the sphere below the dotted line. An estimate for the volume of this curved portion would be a cone with the same base as the large earlier cone with an altitude of $(1-1/\sqrt{2})$*radius. Using this approximation, the volume of the balloon would be $\pi*\text{radius}^3[(4/3)+(1/3)(1/\sqrt{2})^3-(1/3)(1-1/\sqrt{2})]= \pi*\text{radius}^3(1.354)=4.252\ \text{radius}^3$. Therefore $100{,}000=4.252\ \text{radius}^3$ yields a radius of 29 feet.



Junior Varsity Guide Sheet only for those who are not competing for a spot at State. If you have received this **incorrectly**, immediately raise your hand, tell your proctor, and **don't read beyond this point**!

A. Look at how to get the units to cancel and end with the desired units.

B. Use the result in part A.

C. The weight of the displaced air is equal to the weight of the entire balloon system.

D. Remember how to find the equation of a line.

E. Inversely proportional implies that a set of paired values always have the same product. Find the product, write the equation, and substitute the Fahrenheit to Kelvin formula.

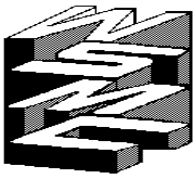
F. Use you answer to part E.

G. Find the total weight of the balloon and compare it to the weight of the displaced air on Mt. Everest.

H. The formula for the volume of a sphere is $(4/3)\pi r^3$, the formula for the volume of a cone is (1/3) area of the base times the height, use Pythagorean theorem to find the diagonal of a square, and be sure to devise and method to approximate the volume (and therefore the radius) **and** explain your reasoning.

Make sure your label your work and make it easy to follow.

Good Luck.



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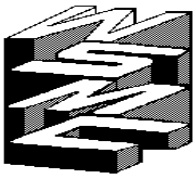
Special answer sheet for Junior varsity teams.



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