

WSMC High School State Competition

Maximizing the Company's Profits

Team Problem

April 18, 2009

An electronics company has put 40 million dollars into research, development, production set-up, and advertising for a new highly desirable product. Its marketing department has used its experience on previous similarly desirable products to project the number of units (in 100,000 units) that will sell at incremental price increases. Table A lists these predictions.

Table A – Predicted Relationship Between the Number Sold and the Price for a Desirable New Product.

Units(100k)	113	111	108	102	89	58	27	14	8	5	3	2	1	0
Price(\$)	0	20	40	60	80	100	120	140	160	180	200	220	240	260

The production department believes they will produce the units for a per-unit cost of \$40. Remember profit is revenue minus **total** costs; revenue is the income from sales, and **total** cost is the sum of the fixed costs and variable costs.

- Create a mathematical model (Price as a function of the number of 100k units) that best represents the data in the table.
- How many units would sell at a price of \$130/unit?
- Create a model for profit and predict the profit for a price of \$140?
- Determine the price that will maximize the company's profit and find the maximum profit. Use diagrams, symbols, and/or words to explain your process for finding the maximum profit and the associated price.
- At the price in part D, how many units must sell for the company to break even (revenue equals the costs)?

Be sure to show all of your work. Organize and label the sections of your work including your data and diagrams.

Scoring Rubric Summarized

Solve Problems using – Measurement – Algebraic Sense – Number Sense

- 8 points – Determines the best model for the data
- 3 points – Determines the number of units that are projected to sell
- 4 points – Determines the profit
- 4 points – Determines the price that maximizes the profit using a valid method and conveys the process
- 3 points – Determines the break even for the answer in part D
5 points – Communicates clearly all work

Maximizing Profits - Rubric for Scorers Team # _____ School _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated appropriately (but of course the answer is incorrect) points for the calculations should be awarded.

A. Determines best model for table (8 points)

Does one of the following:

7 pt – determines the tangent or equivalent model

If another model is generated, compare to the quality of the above to score:

5 pt – uses a cubic regression or equivalent to determine a model

3 pt – uses a linear regression to determine a model

2 pt – determines a model not as good as the previous models but does produce some sort of reasonable prediction

And

1 pt – uses correct values in the model

B. Determines projected unit sales for \$130 (3 points)

1 pt – uses some form of interpolation **OR**

2 pt – uses the model from part C

1 pt – finds projected unit sales

C. Determines profit for \$140 price (4 points)

1 pt – chooses the correct price and number of units

2 pt – creates a model for profit

1 pt – finds the profit

D. Determines maximum profit (4 points)

1 pt – clearly shows process for finding the maximizing values

1 pt – the processes are valid

1 pt – determines a price that gives the maximum profit based on previous work

1 pt – determines the maximum profit based on previous work

E. Determines break-even price (3 points)

2 pt – uses method that can correctly determine the break-even price

1 pt – determines the correct break-even price based on previous work

Overall (5 points) Communication

1 pt – well organized and easy to follow

1 pt – used diagrams and/or good explanations to make clear the solutions

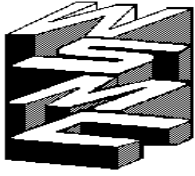
1 pt – mathematical manipulations are shown correctly

2 pt – consistently shows units correctly

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Total Points

29 points possible



Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names _____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

Sample Solution

A. Using price in dollars and number of units in 100,000.

If a linear regression is used:

$$\text{Price} = -1.63(\text{number of units}) + 205. \text{ or } (\text{number of units}) = (205 - \text{price}) / 1.63$$

If a cubic regression is used:

$$\text{Price} = -0.00083(\text{number of units})^3 + 0.15(\text{number of units})^2 - 8.30(\text{number of units}) + 235.5$$

If a tangent model is used (the best model):

$$\text{Price} = -20 \tan(((\text{number of units}) - 58) / 40) + 100$$

B. The simple answer is 2.05 million just using linear interpolation. A linear model predicts 4.58 million. A cubic model predicts 1.78 million. The best fitting model predicts 1.87 million.

C. Profit = Revenue – Costs = $\$140(\text{number of units}) - \$40,000,000 + \$40(\text{number of units})$

$$\text{Costs} = \$40,000,000 + \$40(1,400,000) = \$96,000,000$$

$$\text{Revenue} = \$140(1,400,000) = 196,000,000$$

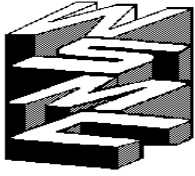
$$\text{Profit} = \$100,000,000$$

D. If the profit is based on the:

- The table: price of \$80 yields a profit = $-400 - 40(89) + 80 * 89 = 3160$ or a profit of \$316,000,000.
- The linear regression of the table: units = 5.05 million, price = \$122.31 for a profit of \$375,700,000
- The cubic regression of the table: units = 8.31 million, price = \$99.10 for a profit of \$451,400,000.
- The tangent regression of the table: units = 7.74 million, price = \$89.46 for a profit of \$342,800,000.

E. Solving $400 + 40x = x * p$ or solving for x , $x = 400 / (p - 40)$ where x is the number of units in 100,000 and p is the price,

- The table yields: $400 / (80 - 40) = 10$ or 1,000,000 units.
- The linear model yields: $400 / (122.31 - 40) = 4.9$ or 490,000 units.
- The cubic model yields: $400 / (99.10 - 40) = 6.8$ or 680,000 units.
- The tangent model yields: $400 / (89.46 - 40) = 8.1$ or 810,000 units.



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A large grid of 20 columns and 30 rows of small dots for writing answers.