

WSMC High School Regional Competition

**Contaminated
Team Problem
March 10, 2010**

A scientist was working late; later than she should have been as she was losing focus. She had been developing a new medicinal drug for a cure to a serious disease. She had several expensive samples of the drug in different vials. In her drowsy state, she might have contaminated one of the samples with a bacteria sample. (She is sure there was no more than one vial contaminated.) The dilemma that you need to help her with is how to determine which vial (if any) is contaminated.

Additional important information:

1. The test for contamination requires placing a drop of the contaminated solution into a Petri dish with a very expensive medium for the bacteria to grow on. The Petri dish with the medium may be used for only one test.
2. It takes 12 hours for the test to yield results.
3. Drops of the medicinal drug may be mixed with each other and if any of the contaminated drug is in the drops placed in the Petri dish, the test will give a positive test for contamination.
4. The results need to be determined as quickly as possible as the drug has a five-day shelf life and she has an important testing meeting scheduled before the expiration date.

You are to work with these two possible scenarios:

Case I: She has only two available Petri dishes with the growing medium. What is the maximum number of samples she could test for contamination?

Case II: She has 60 vials to test. What is the minimum number of Petri dishes needed to complete the testing?

For both Case I and Case II you must:

- A. Develop and describe a strategy that would positively and uniquely identify the contaminated vial (if any).
- B. Use strategies for identification that can be completed in less than five days.
- C. Identify the maximum for Case I and the minimum for Case II.
- D. Link the results in part C to your strategy.

Scoring Rubric Summarized (Case I/Case II)

Solve Problems using – Reasoning – Number Sense

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| A. 3/3 points – A valid strategy is described for each case. | 4/5 points – Results in each case are optimal. |
| B. 2/2 points – The strategy uses less than five days. | 2 point – The answer sheet is well organized and easy to follow. |
| C. 1/1 points – The strategy maximum/minimum is clearly identified. | 2 points – The strategies are clearly described by the use of good grammar, spelling, labeling, charts, diagrams, etc |
| D. 2/2 points – The maximum/minimum clearly identified is well supported by the strategy and explanation. | |

A. A valid strategy is described for each case (3/3 points).

Case I Case II

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- 3 pt – A valid strategy is described **or**
- 2 pt – A strategy is described that has potential but has possibilities of a flaw without assumptions **or**
- 1 pt – A strategy is described that has flaws

B. The strategy uses less than five days (2/2 points)

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- 2 pt – Uses 12 hours
- 1 pt – Uses less than five days but more than 12 hours

C. Max/Min is identified (1/1 points)

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- 1 pt – Maximum (Case I) or minimum (Case II) is identified

D. Max/Min is supported by the strategy and the explanation (2/2 points)

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- 2 pt – Max/Min is supported by the strategy and explanation
- 1 pt – Max/Min is supported by the one or the other

The results for the max/min are optimal (4/5 points)

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- 5 pt – -----/min is six
- 4 pt – Max is three/min is seven
- 3 pt – Max is four/min is seven to fifteen
- 2 pt – Max is two/min is sixteen to thirty (if that is possible)

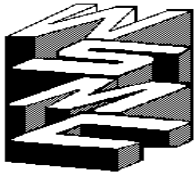
Overall Communication (4 points)

- 2 pt – well organized and easy to follow
- 1 pt – a struggle to decipher
- 2 pt – used diagrams, labels, and good explanations to make clear the solutions
- 1 pt – leaves one or more of these options out.

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Total Points

- 29 points possible
- 22 points is the benchmark for automatic qualification for state for Division I schools and 20 for Division II schools.



Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names _____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

Optimum Solution:

Case I

	V1	V2	V3
P1	x		x
P2		x	x

Put a drop of Vial 1 in Petri 1

Put a drop of Vial 2 in Petri 2

Put a drop of Vial 3 in both Petri dishes

If P1 tests positive, V1 is contaminated

If P2 tests positive, V2 is contaminated

If both test positive, V3 is contaminated

If there are no positive tests there was no contamination.

The maximum is 3 and the vial

Case II

You will need six Petri dishes:

Vials	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Petri 1	x						x	x	x	x	x										
Petri 2		x					x					x	x	x	x						
Petri 3			x					x				x				x	x	x			
Petri 4				x					x				x			x			x	x	
Petri 6					x					x				x			x		x		x
Petri 6						x					x				x			x		x	x

The table shows how to add drops in the Petri dishes. It shows all of the possible ways a drop from the vials can be used. Then it shows using two drops in different Petri dishes from a single vial, then three would be next and then four, then five and six. So if only one Petri dish tested positive, it must be from one of the first six vials. If two Petri dishes test positive, it must be from one of vial 7 through vial 21. Know which two Petri dishes tested positive would tell you which vial was contaminated. The process would continue and you would actually be able to identify the contaminated vial out of 63 contaminated ones. So 6 Petri dishes is the minimum and the test could be done in 12 hours.

Some students may do case two by counting by the binary system:

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20
P1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
P2	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
P3	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1
P4	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
P5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
P6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Where a 1 means a drop was added from the vial into the Petri dish. The combination of the contaminated Petri dishes will predict the contaminated vial. This method is essentially the same as above which uses combinatorics or the binomial theorem.

A less efficient method that might be suggested would be to place a drop from half of the vials into one dish and do the 12 hour test. Then if that half was positive, test half of them. If not test the other half. This process could be repeated. The First Petri dish would reduce the possibilities to 30. The second to 15, the third to 8 (7), the fourth to 4 (3), the fifth to 2 (1), and the sixth to 1 (done). Then that one would have to be tested if the positive one hadn't been tested yet...remember there may not be any that are contaminated. This is a decent method and should score higher than one for one testing but not as high as above.