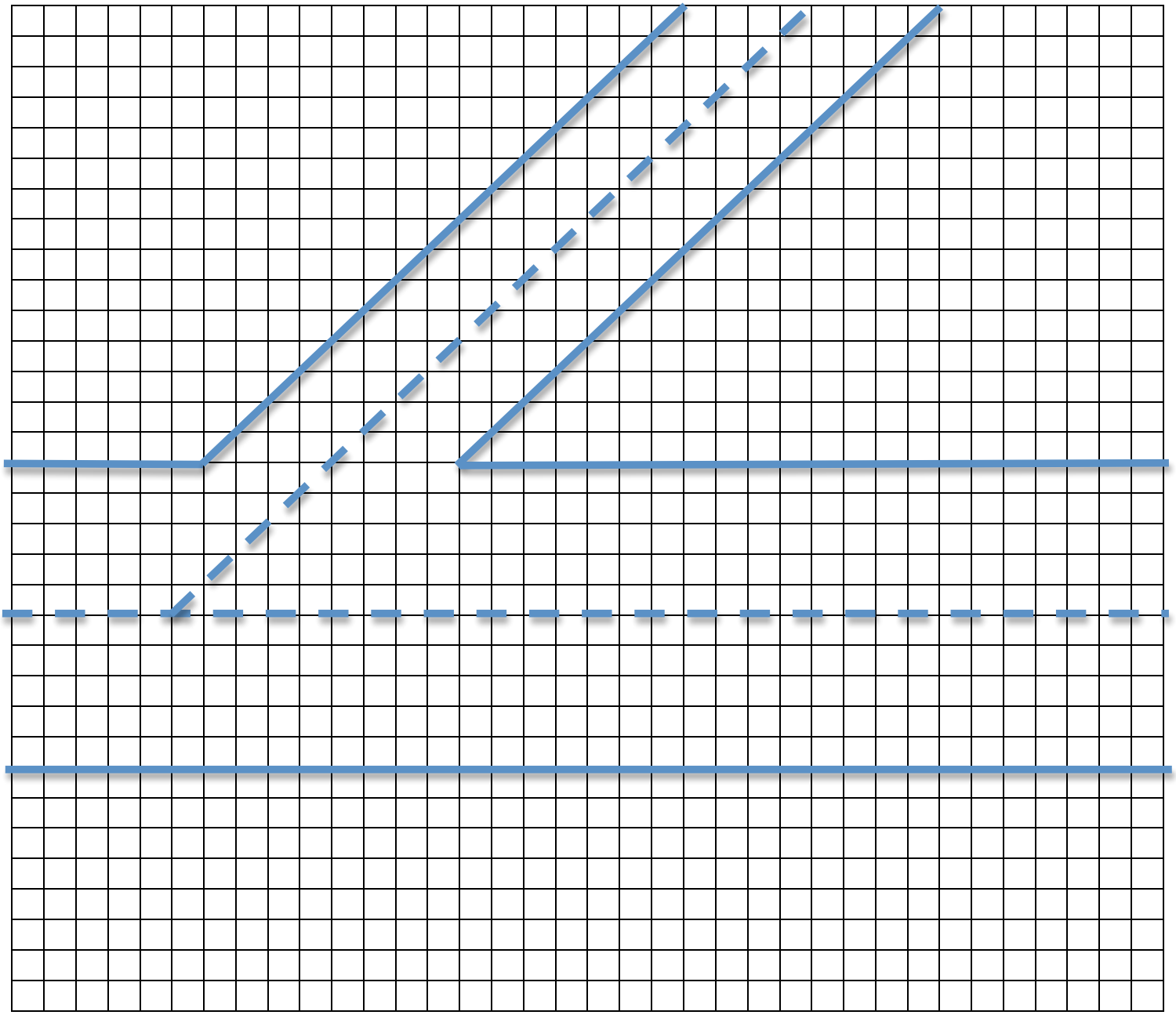
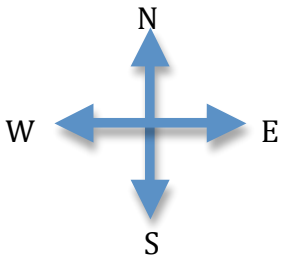


Scale: Each square has dimensions of 10 feet by 10 feet.
Legend: The dashed line is the center of a road and the solid lines are the edges of the roadways.



WSMC High School State Competition

Construction of a Network System
Team Problem
April 17, 2010

A company owns land and buildings all around the intersection of a main roadway and a smaller roadway. It is planning on installing an underground communications network between the three buildings located on the different areas. The map (on the back) shows the multi-lane main roadway headed in the East-West direction. A smaller road leaves this main road headed in the Northeasterly direction. The main roadway is 100 feet wide. (Each square on the map is 10 feet by 10 feet.) To help with descriptions on this problem, assume that the origin of a grid is at the intersections of the centerline of the two roadways and the positive x-axis heads easterly and the positive y-axis heads northerly. Using this grid system, the equations for the edges of the smaller roadway are y=x-40 and y=x+40.

Building A is located northwest of the origin but more importantly, the location of the communications connection point to building A is directly north of the origin at a distance of 200 feet or at the point (0, 200). To the south of the main roadway is building B. Its connection point is 100 feet east of the origin and 100 feet south of the origin or at point (100, –100). Building C's connection point is at (300, 160). Because of city ordinances and the high costs of construction, all utilities built under the roadways have to be perpendicular to the roadway.

A. Determine the length of the shortest distance meeting the requirements for the network between buildings A and C. Include the coordinates of the points on either side of the roadway crossing.
B. Determine the length of the shortest distance meeting the requirements for the network between buildings B and C. Include the coordinates of the points on either side of this roadway crossing.
C. If a junction box was placed at (90, 50) and lines were run from the junction box to A and B and C, determine if this would create a shorter total network installation than construction of the shortest line from A to C and B to C (always meeting the requirements). Show a verification of your answer mathematically.
D. If the junction method is preferred by management, is there a better location (a minimum) along the southeastern edge of the smaller road that would result in even less total network construction distance? Show a verification of your answer mathematically.

Be sure to show your work, demonstrate your reasoning, and include units.

Scoring Rubric Summarized
Solve Problems using – Reasoning – Number Sense

A. **5 points** – Determines the shortest distance between A and C and the crossing points

B. **6 points** – Determines the shortest distance between Band C and the crossing points

C. **7 points** – Determines each distance to the junction point and the total distance.

D. **7 points** – Develops a strategy for finding the location for junction that includes a minimum, determines the location, total and how it compares to part C.

4 point – The answer sheet is well organized and easy to follow with proper units and labeling.

Network Score Sheet Team School _____ Names _____

A. Determines minimum for A to C (5 points).

	2 pt – Determines horizontal is best on parcels and perpendicular across road **or** 1 pt – Uses an estimation that isn't valid for the minimum but is close
	2 pt – Determines points at edge of roadway correctly for method chosen above (1 pt each)
	1 pt – Determines total length

B. Determines minimum from B to C (6 points)

	3 pt – Determines the appropriate direction for the network line for the portion not under the roadway by a valid method for producing the minimum **or** 2 pt – Appears to estimate the minimum with some understanding/minor errors **or** 1 pt – Draws a straight line to find a point in the center of the roadway and uses that to derive an answer.
	2 pt – Determines the points on either edge of the roadway (1 pt each)
	2 pt – Determines the total length

C. Determines the distances for junction point (90, 50) (7 points)

	2 pt – Determined the minimum distance from junction point to A **or** 1 pt – Determines a reasonable estimate that isn't quite correct
	2 pt – Determined the minimum distance from junction point to B **or** 1 pt – Determines a reasonable estimate that isn't quite correct
	2 pt – Determined the minimum distance from junction point to C **or** 1 pt – Determines a reasonable estimate that isn't quite correct
	1 pt – Determined the total distance based on the three above

D. Determines the junction point for the minimum (7 points)

	3 pt – Designs a method that could determine the minimum **or** 2 pt – Designs a method that has minor flaws **or** 1 pt – Designs a method with major flaws
	2 pt – Determines the coordinates for the minimum based on method above (1 pt each coordinate
	1 pt – Determines the total distance based on the above
	1 pt – Indicates whether this minimum is better than part C

Overall Communication (4 points)

	1 pt – well organized and easy to follow
	1 pt – ideas and concepts are clear and well written
	2 pt – Consistently uses units

	Total Points 29 points possible

Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names_____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

A. Connecting A to C by the shortest distance is easiest to see by translating A (or C) to eliminate the effect of having to move perpendicular to the roadway. Translating A to the coordinates (40, 160) shows that the shortest distance between A and C would run east-west except for the road. The distance from (40, 160) and (300, 160) is 260 feet. Crossing the roadway is would then be from (160, 200) to (200, 160) which is $40\sqrt{2}$ feet or 56.6 feet. So the total distance from A to C would be 316.6 feet.

B. Using a similar technique and translating B 100 feet north would put B at (100, 0). A straight line from this point to C (300, 160) would have the equation of $y=(4/5)x-80$. The intersection of this line and $y=50$ (the northern edge of the main roadway is at the point (162.5, 50). Therefore the line from C (300, 160) would go to point (162.5, 50), cross the main roadway to (162.5, -50) and then on to B (100, -100) which is a total distance of:

$$\sqrt{((300-162,5)^2+(160-50)^2)} + 100 + \sqrt{((100-162.5)^2+(100-50)^2)} = 356.1 \text{ feet.}$$

C. The distance from the junction to A is:

$$\sqrt{(50^2+110^2)} + 40\sqrt{2} = 177.4 \text{ feet}$$

The distance from the junction to C is:

$$\sqrt{(210^2+110^2)} = 237.1 \text{ feet}$$

The distance from the junction to B is:

$$\sqrt{(10^2+50^2)} +100 = 151.0 \text{ feet}$$

Therefore the total distance for this network is 177.4+237.1+151.0 = 565.5 feet. This is shorter than 356.1+316.6 = 672.7 feet or 107.2 feet shorter.

D. If we remove the effect of the roadways by translating A to (40, 160) and B to (100, 0) and then add in the roadway widths, and the point along the SE edge of the smaller roadway is at a point (x, x-40) then the distance of the total network is expressed by:

$$\sqrt{((40-x)^2+(160-(x-40))^2)} +40\sqrt{2} + \sqrt{((300-x)^2+(160-(x-40))^2)} + \sqrt{((100-x)^2+(0-(x-40))^2)} + 100$$

Entering this expression and minimizing it yields x = 124.3 feet. The y coordinate is x-40 or 84.3 feet. The total network is 548.9 feet. This is even shorter than the distance in part C.

There are numerous other ways to solve this problem.

Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names_____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.