

WSMC High School State Competition

# Topical Problems

## Part One

April 17, 2010

**Directions:** Mark on the answer form the letter that is closest to the correct answer. Make sure that your name(s) are on the answer form. If you are on a team put your team number and school name on the answer form. Remember that there is 5 points awarded for a correct response, 1 point for no response, and 0 points for an incorrect response.

In the year 2000 there were 47,368 births in Utah and 13,603 births in Maine. The populations in those two states were approximately  $1.8 \times 10^6$  and  $1.3 \times 10^6$ .

- How many people were born per hour in Maine in 2000?  
A. 1.6      B. 3.1      C. 37.2      D. 5.41
- What was the birth rate in births per 1000 people in Utah in 2000?  
A. 47      B. 26      C. 36      D. 14
- The state with the greatest birth rate in percentage of population was how much greater than the other state's birth rate percentage?  
A. 1.6      B. 3.1      C. 0.8      D. 5.41

Two math teachers are teaching a unit on solving equations. Each teacher creates his/her own test for the subject. Assume that they have taught the material very similarly and if they had given the same test to both classes, they would have gotten very similar scores. However they gave their own tests to their own classes with the results shown in the table.

|           |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|-----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Teacher A | 75 | 82 | 67 | 90 | 95 | 35 | 73 | 73 | 75 | 84 | 88 | 87 | 63 | 82 | 90 | 90 | 72 | 71 | 85 | 80 |
| Teacher B | 85 | 81 | 86 | 84 | 83 | 80 | 85 | 82 | 81 | 81 | 84 | 80 | 82 | 85 | 81 | 82 | 85 | 83 | 82 | 81 |

- Which teacher's test seems to have the highest percentage of easier problems?  
A. Teacher A      B. Teacher B      C. About the same
- What is the difference in the median scores of the two tests?  
A. 2      B. 8      C. 11      D. 5
- What is the difference in the standard deviation between the two tests?  
A. 11.5      B. 3.1      C. 37.2      D. 5.4
- If the outlier in Teacher A's scores was removed, and the mean, mode and median were re-calculated, there would be changes. Noting the changes and ranking these changes in the measures of 'average' from greatest changed to least changed, which reflects that ranking? (Using  $\Delta$  to mean change)  
A.  $\Delta$  mode  $>$   $\Delta$  mean  $>$   $\Delta$  median      B.  $\Delta$  median  $>$   $\Delta$  mean  $>$   $\Delta$  mode  
C.  $\Delta$  mean  $>$   $\Delta$  mode  $>$   $\Delta$  median      D.  $\Delta$  mean  $>$   $\Delta$  median  $>$   $\Delta$  mode

The table shows the record for the most home runs in a single season since 1885.

| Player         | Year | HR |
|----------------|------|----|
| George Hall    | 1876 | 5  |
| Charley Jones  | 1879 | 9  |
| Harry Stovey   | 1883 | 14 |
| Ned Williamson | 1884 | 27 |
| Babe Ruth+     | 1919 | 29 |
| Babe Ruth+     | 1920 | 54 |
| Babe Ruth+     | 1921 | 59 |
| Babe Ruth+     | 1927 | 60 |
| Roger Maris    | 1961 | 61 |
| Mark McGwire   | 1998 | 70 |
| Barry Bonds    | 2001 | 73 |

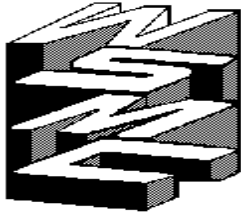
8. If the data is modeled by a linear function, how many home runs will be hit in the 2020 season as predicted by the model?  
 A. 80      B. 85      C. 90      D. 95
9. If the data is modeled by a quadratic, what is the predicted maximum number of home runs to be hit in a single season?  
 A. 77      B. 71      C. 80      D. 74

Consider an isosceles trapezoid with one of the bases with a length of 15 cm, the other base is 'b' and an area of the trapezoid is 100 square cm.

10. If the other base ('b') is 12 cm, what is the height in cm?  
 A. 6      B. 7      C. 8      D. 9
11. Which formula would express the relationship between the base 'b' and the height 'h'?  
 A.  $15+b+2h=100$       B.  $100/h=15+b/2$       C.  $200/h-15=b$       D.  $200+15h=bh$
12. If the legs are 8 cm each, what is the length of base 'b'?  
 A. 9.5      B. 10.5      C. 10.0      D. 11.0
13. What length in centimeters of base 'b' would minimize the perimeter?  
 A. 8.9      B. 0      C. 6.5      D. 15.0

Consider the sequence 1, 1, 1, 3, 5, 9...

14. The next terms can be predicted by which of the following:  
 I  $a_{n+2}=a_n + a_{n+1}$   
 II  $a_{n+3}=a_n+a_{n+1}+a_{n+2}$   
 III a polynomial function of degree 5 where  $f(1)=1, f(2)=1, f(3)=1, f(4)=3, \dots$   
 A. I      B. II      C. III      D. two of the three choices
15. If the seventh term is 17 and the eighth term is 31, what would the ratio of any term divided by the term before tend to be as the terms get very large?  
 A. 1.9      B. 1.7      C. 1.8      D. 2.0



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# Topical Problems

## Part Two

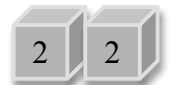
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Suppose there is a parcel of land with the following description: Starting from corner P the property line extends 300 feet to the east to corner Q then the property line extends 260 feet north to point R then westward to point T 300 feet from point Q then following an arc with center at Q back to point P.

1. What is the length in feet of the property line from point R to point T?  
A. 170      B. 160      C. 140      D. 150
2. What is the measure of the angle PQT in degrees?  
A. 30      B. 40      C. 50      D. 60
3. What is the area in square feet of the parcel?  
A. 66,600      B. 43,000      C. 67,000      D. 85,000
4. What is the perimeter of the parcel?  
A. 935      B. 767      C. 1024      D. 867

Two cubes have digits on them so that the day's calendar date can be displayed. The figure shows a few examples: the 3 day of the month, the 17<sup>th</sup> day of the month, and the 22<sup>nd</sup>. Every date is shown with two digits.



5. How many digits need to be on both cubes?  
A. 3      B. 1      C. 2      D. 0
6. How many distinct ways are there to split the digits into two groups where one group is the digits placed on one cube and the second group on the other cube?  
A. 2      B. 20      C. 6!      D. 6

The market research department produced the table relating price and demand for their newest product. The production department projected that the following function gives the cost of producing a single unit:  $C(x)=500,000 + 15,000x$ . The variable  $x$  is in thousands of units ( $x=1$  means one thousand units produced). The company figures that it needs to markup their cost by 40% to cover overhead including marketing, distribution, etc.

| Price - Demand Table     |                     |
|--------------------------|---------------------|
| Price Per Unit (dollars) | Demand (1000 units) |
| 30                       | 1000                |
| 35                       | 850                 |
| 40                       | 700                 |
| 60                       | 100                 |

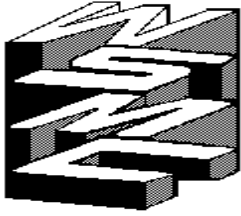
- What is the total cost (cost plus markup) in millions of dollars for producing 1,000,000 units?  
A. 15      B. 20      C. 6      D. 50
- What is the net profit in millions of dollars for producing 1,000,000 units?  
A. 5      B. 10      C. 1      D. 20
- What price would yield the maximum net profit?  
A. 35      B. 55      C. 40      D. 48

Consider the coordinate plane drawn on a piece of graph paper. Point P has coordinates  $(-3, 2)$ , point Q has coordinates  $(5, -2)$ . R is the point where segment PQ intersects the y-axis.

- What is the distance between P and R?  
A. 3.4      B. 3.3      C. 3.2      D. 3.1
- What is the slope of the line perpendicular to line PQ?  
A. 0.5      B. 1      C. 1.5      D. 2
- If the paper that the coordinate plane is drawn on is folded along the y-axis so that the first and second quadrants are on top of each other as are quadrants three and four, what is the measure in degrees of angle PRQ?  
A. 53      B. 126      C. 63      D. 60
- If the folded paper in number 12 is unfolded half-way so that the plane containing quadrants I and IV is perpendicular to the plane containing quadrants II and III, what is the measure of angle PRQ in degrees? (Note that P, Q, and R are no longer collinear and you are dealing with a three dimensional situation.)  
A. 99      B. 96      C. 90      D. 93

The world's population was estimated to be 3 billion in 1960. The growth rate peaked at 2.1% per year at that time. Assume that the rate of growth depends on the carrying capacity of the earth by the relationship  $R_A = R(1-P/C)$  where  $R_A$  is the adjusted growth rate,  $R$  is the current rate of growth,  $P$  is the population, and  $C$  is the carrying capacity.

- Assume the carrying capacity of the earth is one person per acre of land and the earth has land area of about 25% of its surface. The radius of the earth is 4000 miles and one square mile is 640 acres. What is the carrying capacity in millions of persons of the earth based on these assumptions?  
A. 5000      B. 20,000      C. 10,000      D. 30,000
- If experts consider other factors and determine the carrying capacity of the earth to be 12 billion people, what is the rate of growth predicted for 1960?  
A. 1.5%      B. 1.0%      C. 2.0%      D. 0.5%



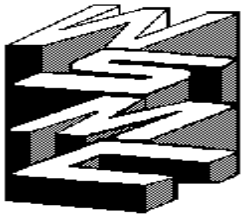
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## Answer Key

April 17, 2010

| Part One |              | Part Two |               |
|----------|--------------|----------|---------------|
| 1.       | A 1.55       | 1.       | D 149.7       |
| 2.       | B 26.3       | 2.       | D 60.1        |
| 3.       | A 1.59%      | 3.       | A 66663       |
| 4.       | B Nearly 60% | 4.       | C 1024        |
| 5.       | A 1          | 5.       | A 3 (0, 1, 2) |
| 6.       | A            | 6.       | B 20          |
| 7.       | D            | 7.       | B 21.7        |
| 8.       | C 89.5       | 8.       | B 8.3         |
| 9.       | B 71         | 9.       | C 42.17       |
| 10.      | B 7.4        | 10.      | A 3.354       |
| 11.      | C            | 11.      | D 2           |
| 12.      | D 10.87      | 12.      | A 53.1        |
| 13.      | A 8.9        | 13.      | A 101.5       |
| 14.      | D            | 14.      | D 32200       |
| 15.      | C ~1.84      | 15.      | A 1.575       |



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