HIGH SCHOOL CORNER (9-12)

In the last issue of Washington Mathematics Journal, the editor took the liberty of editing part one of "Developing the Concept of the Linear Model" by Jim Miller and would like to retract editing errors in paragraphs 5 and 6. Corrections to those two paragraphs are at the right.

*Events at reading at the article by Jim Miller has brought up some interesting discussion among math teachers. When looking at points that determine patterns, more than two points are needed to determine a pattern. However, this article is about **linear** patterns where two points are sufficient.*

… Donna Buck

Linear Patterns

In a sequence of numbers, each number is called a term. (For the sequence 2,5,8,11, … etc. the first term is 2, the second term is 5, etc.) **The gap between the numbers is called the difference.** (In this example the difference is 3, because 5-2=8-5=11-8=3.) **Linear patterns have the same difference between the number of the term and the term itself. The rule is the same for each term.** (In this example 2=3x1-1, 5=3x2-1 and 8=3x3-1, so the rule is t=3xn-1 or t=3n-1)

Linear patterns repeat indefinitely in either direction along a line. Beads on a necklace, walking footprints, musical rhythms, the meter of poetry are all examples of linear patterns that are created or extended by the regular repetition of units, sounds, or events.

A linear pattern is said to exist when the points examined form a straight line.

Developing the Concept of Linear Model Part One

Corrected Paragraphs 5 and 6 of Part 1

x	y
2	5
3	7

To scaffold on what the students have learned, let's start by looking at patterns. Students are quite good at patterns and I have learned to appreciate the elementary teachers and their hard work at building these skills. (It is unfortunate that often teachers don't scaffold on the skills taught students.) After all, the linear model is nothing more than at description of a pattern. Take for example the table (T-chart or whatever you want to call it). Ask the students what is going on in the x column. Without hesitation, they are quick to point out that it is going up by one. Ask them what the next two values would be. Again, they find this an easy task (thanks to their elementary teachers). Proceeding to the y column we get similarly quick responses. Now reflect on this question: Given point A(2, 5) and point B(3, 7) find two more points that are on the line passing through these two points? We have just answered this question probably more easily than the many ways that could and probably would have been used.

Then asked the students to graph the two points? Of course they need to be familiar with the Cartesian coordinate plane and plotting points on it. Sure there are the usual problems: x and y coordinates mixed up, plotting each coordinate as a separate point, etc. Students need to be helped through these common errors. There is a more important problem that may cause the students to develop misunderstandings. Consider how often with only two points and students, not being very good at spacing the first unit or two on the graph in both the x and y direction, get what appears to be widely varying slopes for the same two points. This is a good time to discuss this problem but it still doesn't go away. When using the table, however, they quickly generate a couple of extra points and the slopes tend to be much more accurate.

DEVELOPING THE CONCEPT OF THE LINEAR MODEL

Developing the Concept of the Linear Model
(Part Two)

By Jim Miller

Cle Elum-Roslyn High School
Cle Elum-Roslyn School District

In Part One, we learned how to use tables to develop the linear model equations using constructivism. Now we shall take that critical step of applying that knowledge to real situations. For this, I have developed a worksheet. In doing this next section, I discovered many wonderful un-intended concepts being taught that made the mathematics rich, powerful, and interesting.

First, when students work with the linear model in the real world situations, the information usually is less than perfect. Most data has some error. Therefore, to deal with real data and to use the methods just learned, students are more successful if they graph the data, estimate the line of best fit visually, pick two representative points and then develop the linear model. Organizing these steps for the student (and for the teacher that reviews their work) is the point of the 'Data and Analysis Worksheet.'

Before we actually collect data, let's highlight the important features of the worksheet. Notice that in the upper left had corner is an x, y table. This is where their raw data is placed. Next they are expected to graph the data on the grid in the lower left. This section turns out to be a great learning experience for the students. Most students have no trouble graphing the sterile examples we've worked with up to now, but when real data is thrown in, scaling, units, spacing, labeling, and sizing became a major challenge. Consider spending a great deal of time helping students with various strategies for graphing real data. It pays in the long run.

Once they finally get the graph, it again is a challenge to get the students to draw a representative line to fit the data. Commonly they try to connect dot to dot or draw a seemingly random line. Try to get them to think of the points as a trail. Push them to think of the straightest trail that would follow the same path as the points, 'the tracks.' This also is surprisingly difficult for the students to do but is a fundamental concept of modeling and understanding error.

Finally, another challenge was to get the students to understand what 'picking two representative points' means. When they finally are able to pick the two points, the points' coordinates need to be entered in the x, y table to the right of the original data. Using the representative data and the spaces around this second table, I have the students develop the linear model both in the general form and in the slope-intercept form. By now this process should be familiar to them. This much should seem like practice to the students and doing several different labs will develop their skills.

Notice that there are sections of the worksheet that aren't being used. Let me explain their purpose and consequently where I go from here. One of the labs I do early on will have a y-intercept of zero. Ask the students to notice what happens in this case when either the original data points or the representative data points' coordinates are divided. Then I discuss that in this situation they are dealing with a special linear model called direct variation. Hence there is the section on the worksheet to the right of the table of the raw data for finding the direct variation constant developing the direct variation model. The top right is a section for finding the inverse variation constant and inverse variation model. When students suspect that their data represents an inverse relationship, they should rewrite their raw data in the x and y columns and calculate x*y. If the x*y column is fairly constant, they can write the model x*y = (average if the x*y column).

The other two sections make this a very powerful unit and really help the students understand the power of the linear model. Suppose that students collected some data that wasn't linear. Let's say that it was quadratic symmetric with the y-axis. Some of the labs presented later have that have this characteristic. Before calculators made finding quadratic models a matter of pushing buttons, curve straightening techniques were used. Curve straightening and quadratics usually are not Algebra I topics but are a natural way to practice using the linear model. They also add richness to the unit. There is very little additional material needed to reach this higher level of understanding of the importance and power of the linear model.

If the students recognize that the data is quadratic, which requires spending time discussing quadratic, cubic, square root and inverse models, then they create a new table in the center of the page to the right of the graph. In this table, they enter their original data, square the x values, and graph x^2 versus y. Encourage the students to use a new sheet of graph paper as the new graph may not fit well on the graph at the left. If their guess that the data is quadratic is correct, the new graph should be relatively linear and they should derive the linear model using representative points in the table at he top of the work sheet. Once they get either the general form model or the slope-intercept model, have them replace the x variable with x^2. They now have a quadratic model for the original data.

The table in the lower right is for other possible relationships and can be calculated like the quadratic model was just above it.

On the back of the worksheet are questions that ask the students to consider what their domain and range are. There is very little insight into domain and range for linear models unless the context for the data is included. With the context, these become rich discussion questions.

Now let's get started with some of the labs my students are asked to do. You may choose which labs you would like to do. It would be over kill to do all of the labs. Please pick and choose labs that meet the instructional needs of your students. I like to vary the labs I use each year to avoid boredom. The table shows the title of the various labs, which model they exemplify, notes on their difficulty and comments on the domain and range. Each lab is given in a brief form as well as in a detailed form. Use the form that is appropriate for the skill level of your students. Using the shortened version helps the students become independent thinkers but the more detailed form will help them have more success early on.

Developing the Concept of the Linear Model continued ...

Information on the Different Labs

Title	Model	Difficulty	Domain	Range
The Ball Bounce	Linear	Medium-some challenge in taking measurements	0 to 100 cm what the lab asks for but could go as high as 30 meters or so. After that air friction would change the model.	Similar to the domain except the maximum is less than the domain's maximum value.
The Discrete Advertisement	Linear	Easy-just multiply the cost per unit by the number of units	0 to either what a person might conceivably by or the maximum a store might carry. The domain is discrete if the ad is 'so much per package.'	0 to the maximum amount determined in the domain multiplied by the unit price. It may be interesting to not that the range is discrete also.
The Continuous Advertisement	Linear	Easy-just multiply the cost per unit by the number of units	0 to either what a person might conceivably by or the maximum a store might carry. The domain is continuous if the ad is 'so much per pound' and the package size can vary.	0 to the maximum amount determined in the domain multiplied by the unit price. It may be interesting to not that the range is discrete also.
The Teeter-Totter Lab	Inverse	Medium-there is come challenge in taking the measurements	0 to 45 cm because that is the length of the meter stick hanging over the edge of the table and it is continuous.	Near 0 to infinity and continuous.
The Galileo Table Roll	Quadratic	Hard-marking the position of the rolling object in even time units is rather challenging.	Minimum of 0 to a maximum of something like 10 to 20 beats and continuous.	0 to the length of the table, something like 2-3 meters and continuous.
The Circumference of a Circle	Linear	Medium-Students have trouble wrapping the string and measuring it.	0 to very large, limited to the size of a circle a string could be wrapped around, possibly the radius of he earth, continuous.	0 to very large, limited to the size of a circle a string could be wrapped around, possibly the circumference of he earth, continuous.
The Area of a Circle	Quadratic	Medium-Students will have trouble counting the squares and making good circles	0 to size that could be drawn, probably something smaller then a few kilometers or the circle wouldn't be flat-the curvature of the earth begins to be a concern, continuous	0 to size that could be drawn, probably something smaller then a few kilometers or the circle wouldn't be flat-the curvature of the earth begins to be a concern, continuous
The Volume of a Sphere One	Cubic	Hard-Students will be measuring with tools they are unfamiliar with.	0 to large as in a few kilometers. It could be larger but hard to use the methods of this lab to measure, however the rule should apply to larger spheres, continuous.	0 to large as in a few km. It could be larger but hard to use the methods of this lab to measure, however the rule should apply to larger spheres, continuous.
The Weight of Proportional Wooden Cylinders	Cubic	Medium-only because the students may be using a scale for the first time. Harder for the teacher to come up with the cylinders.	0 to large, limited to the size they could maneuver however the domain theoretically goes to infinity, continuous.	0 to large, limited to the size they could maneuver however the domain theoretically goes to infinity, continuous.
The Pendulum Lab	Quadratic/Square Root	Hard-Trouble measuring the time and length to the center of the mass and getting short lengths.	0 to 30 meters. Longer will have all kinds of friction problems, continuous.	0 to something like a 10 seconds. The time doesn't change much as the length gets longer.
The Volume of a Sphere Two	Cubic	Medium-It is a bit of a challenge to make each breath of equal.	0 to 30 breaths. There won't be many balloons that will hold more than that, continuous. The rule would apply to larger.	0 to 50 cm. Again, not many balloons larger than that, continuous. . The rule would apply to larger.
The Volume of a Cylinder	Linear	Medium- It is a bit of a challenge to make each breath of equal.	0 to 20 breaths. There won't be many balloons that will hold more than that, continuous. . The rule would apply to larger.	0 to 50 cm. Again, not many balloons longer than that, continuous. . The rule would apply to larger.
The Surface Area of a Sphere	Quadratic	Hard-Hard to cover approx. squares of equal size.	0 to 30 cm. There won't be many balloons larger than that, continuous. . The rule would apply to larger.	0 to 120 cm^2, continuous. The rule would apply to larger.

Hooke's Law	Linear	Medium-springs are better, but rubber bands give reasonable results.	0 to ???-varies with the mass of the individual weights, continuous.	0 to the maximum length the rubber band can be stretched to, continuous.
Friction	Linear	Medium-challenge to read the scale well.	0 to 4 or 5 books then the scale maxes out, discrete but could be argued continuous.	0 the maximum of the scale, continuous.
Chords in a Circle	Inverse	Medium-trouble making good circles.	0 to 20 cm depending on the size of the circle and the location of the point, continuous.	0 to 20 cm depending on the size of the circle and the location of the point, continuous.
Area of a Rectangle	Inverse	Easy-simple counting.	1 to 48, discrete.	1 to 48, discrete.
Card Lab One	Linear	Hard-estimation & interpretation problems	0 to 10 arm lengths, hard to see after that, continuous.	0 to 200-300 cm, continuous.
Card Lab Two	Inverse	Hard-estimation & interpretation problems	0 to 10 arm lengths, hard to see after that, continuous.	~6 cm to infinity, continuous.
Hands Across America	Linear	Medium-measurement issues.	0 to class size or world population size (may not be possible because of human error), discrete.	Greater than 0 to 10 seconds in a classroom, days in the world situation, continuous.
Heads or Tails	Linear	Medium-only because the data is not very linear for small values.	0 to very large numbers but arguably not infinity, discrete.	0 to very large numbers but arguably not infinity, discrete.
The Toss of a Die	Linear	Medium-only because the data is not very linear for small values.	0 to very large numbers but arguably not infinity, discrete.	0 to very large numbers but arguably not infinity, discrete.
It's All Knotted Up	Linear	Medium-maintaining consistency.	0 to 20-40, discrete,	Length of rope to some fraction of the length of the rope, continuous.
The Balloon Sled	Quadratic???	Hard-challenge to make and be consistent.	Something larger than 0 to 40 cm, continuous.	0 to 20 meters, continuous.
Create Your Own Relationship	Depends on Choice	Depends	Depends	Depends

THE BALL BOUNCE

Materials: Each group will need a meter sticks and a ball. A variety of balls that bounce well can be used. Suggested balls are: ping-pong balls, golf balls, tennis balls, volleyballs, whiffle balls, super balls, marbles, basketballs, etc.

Instructions: Drop your ball from 10 cm, 20 cm, 30 cm… 100 cm. The drop height should be measured from the bottom of the ball to the surface the ball is being bounced off of. You may want to drop the ball from each height several times until your group agrees on the bounce height. Record your results, determine the model, and complete the worksheet.

THE DISCRETE ADVERTISEMENT

Materials: Each group will need to find an advertisement for some product where the domain is will be a discrete value.

Instructions: Using your advertisement, determine the cost for various quantities of your item. Record your results, determine the model, and complete the worksheet.

THE CONTINUOUS ADVERTISEMENT

Materials: Each group will need to find an advertisement for some product where the domain is will be a continuous value.

Instructions: Using your advertisement, determine the cost for various quantities of your item. Record your results, determine the model, and complete the worksheet.

THE TEETER-TOTTER LAB

Materials: Each group will need a meter stick and a significant quantity of similar weights like pennies.

Instructions: Place the meter stick on the edge of a student desk so that 55 cm of the meter stick is on the table and 45 cm of the meter stick is off the desk. Place just enough weights at 5 cm from the edge of the desk to tip the ruler. Record the distance (5 cm) and the number of weights. Repeat at 10 cm form the desk and 15 cm, 20 cm, 25 cm … Record your results, determine the model, and complete the worksheet.

THE GALILEO TABLE ROLL

Historical note: Galileo reasoned that the pattern of distances from an object rolling down a table had to match the pattern for an object falling. His thinking was that since the steeper the slope of the table the faster the pattern happens and eventually when the table is vertical, the pattern would be that of a falling object. This enabled him to predict the motion of a falling object by studying it in 'slow motion.'

Materials: Each group will need an object to roll down a **slightly** sloping table (~2-4 cm per meter). They will also need either a dry erase marker (if the table tops are such a surface that the dry erase marker easily washes off) or a strip of paper like adding machine paper.

Instructions: Have one student practice counting and marking at a steady rhythm of about 2 to 3 marks or beats per second. Another student will release the rolling object and the rhythm student will mark the position of the object at each beat. Measure the distances from the first beat's mark to each of the other beats' marks. Record the number of beats and the corresponding distance. Determine the model, and complete the worksheet.

THE CIRCUMFERENCE OF A CIRCLE

Materials: Each group will need to find 5-8 round objects of varying diameters. The more variation in the diameter the better the results will be. Each group will also need a meter stick and a piece of string long enough to reach around their largest object.

Instructions: Measure the diameter and using the string, the circumference of each of your circles. Record your results, determine the model, and complete the worksheet.

THE AREA OF THE CIRCLE

Materials: Each group will need a ruler, a piece of graph paper, and a compass.

Instructions: Draw varying size circles on the graph paper. Using the side of each square on your graph paper as a unit of one, determine the radius of each circle. By counting using estimation determine the area of each circle in terms of the number of squares in the circle. (A possible method is to count every square that is more than 50% inside the circle as a square and not counting any other squares.) Record your results, determine the model, and complete the worksheet.

THE VOLUME OF A SPHERE ONE

Materials: Each group will need a ruler, a variety of spheres from the size of a Beebe to the size of a soft ball, water, and different sizes of graduated cylinders and or beakers. The graduated cylinders and beakers should be readily available from your science department.

Instructions: Measure the radius of each of your spheres to the nearest millimeter. Then by immersing your spheres in water and noting the change in the water level, determine the volume of each sphere. Record your results, determine the model, and complete the worksheet.

THE WEIGHT OF PROPORTIONAL WOODEN CYLINDERS

Materials: Each group will need a ruler, a scale that measure at least to the nearest gram, and a variety of wooden cylinders similar in shape. The cylinders can be collected by purchasing various size dowels and cutting off pieces whose diameter equals their length. It is best if the pieces of wood are similar in density.

Instructions: Measure the diameter of each of your cylinders to the nearest millimeter. Then weigh your cylinders. Record your results, determine the model, and complete the worksheet.

THE PENDULUM LAB

Materials: Each group will need two rulers, a piece of thread, a weight like a washer, and a timer.

Instructions: Tie the washer to the end of the thread. Wrap the thread around the end of one of the rulers. Measure the distance from the edge of the ruler that the thread is wrapped around to the center of the washer. Measure the time of one period of the pendulum (pull the washer back a few centimeters and time ten complete 'over and back' swings of the washer and divide the time by ten). It is important to get several data points for very short lengths of thread. Record your results, determine the model, and complete the worksheet.

THE VOLUME OF A SPHERE TWO

Materials: Each group will need a ruler and a balloon that is as spherical as possible.

Instructions: This may take some practice but add one breath of air to your balloon. Measure the radius of your balloon. Add another breath. Find the radius for this balloon. Continue finding the radius for each additional breath of air. Record your results, determine the model, and complete the worksheet.

THE VOLUME OF A CYLINDER

Materials: Each group will need a ruler and a balloon like those used by balloon artist to make balloon animals of characters.

Instructions: This may take some practice but add one breath of air to your balloon. Measure the length of the inflated portion of your balloon. Add another breath. Find the new length of the balloon. Continue finding the lengths for each additional breath of air. Record your results, determine the model, and complete the worksheet.

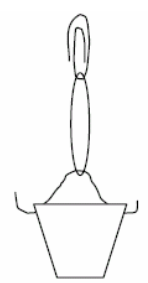
THE SURFACE AREA OF A SPHERE

Materials: Each group will need a ruler and a balloon that is as spherical as possible.

Instructions: Blow the balloon the balloon up to about as large as possible. As best as you can, draw with a vis-à-vis marker, squares of equal size on the surface of the balloon. This will take lots of estimation and approximation. Take your time and do the job well. Measure the radius in centimeters of the balloon and estimate the surface area in cm^2. Release some of the air, calculate the new radius, and surface area. Release some more air and collect more data. Repeat at lease a few more times. Record your results, determine the model, and complete the worksheet.

HOOKE'S LAW

Materials: Each group will need a ruler, a paper cup, a thin rubber band about 4 to 8 cm long, two paper clips, and some uniform weights like pennies.



Instructions: Make a handle on your paper cup using one paper clip. Have the rubber band around the cups handle and the other paper clip as shown. Add varying numbers of weights to the cup and record the length of the rubber band. Record your results, determine the model, and complete the worksheet.

FRICTION

Materials: Each group will need a spring (fish) scale that measures up to ~2000 grams a three foot piece of string and four equal sized text books.

Instructions: Tie the ends of your string together to make a large loop. Slip the loop into about the middle of one of the books. Hook the spring scale to the string and pulling horizontally, determine how much force it takes to start the book sliding. Try it several times at various spots on your desk and use an average value. Stack the second book on the first and determine the force required to slide the two. Then determine the force for three and four. Record your results, determine the model, and complete the worksheet.

CHORDS IN A CIRCLE

Materials: Each group will need a ruler and a compass.

Instructions: Draw a large circle on a piece of paper. Locate a point P a few centimeters from the edge of the circle on the inside the circle. Draw a random chord in the circle that goes through point P. Measure the distance from P to one of the chord and from P to the other end of P. Record the data in the table in the upper left hand corner of your worksheet. Draw a new chord through P and again measure the distances to each end of the chord from P and record your data. Repeat several more times. Record your results, determine the model, and complete the worksheet.

AREA OF A RECTANGLE

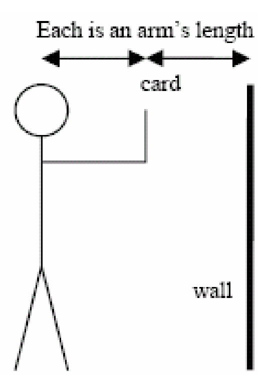
Materials: Each group will need a ruler and a piece of graph paper.

Instructions: Draw a rectangle that has an area of 48 squares. Note the length and with of your rectangle and record the data in the table in the upper left hand corner of your worksheet. Draw a different rectangle that still has an area of 48 squares. Again record the length and width. Repeat several more times. Record your results, determine the model, and complete the worksheet.

CARD LAB ONE

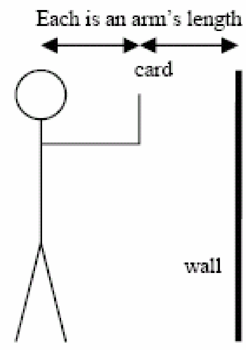
Materials: Each group will need a meter stick and a three by five card.

Instructions: The card holder will face a wall with shoulders parallel to the wall. They will hold the 3x5 card at an arms length from them against the wall. A second person will note the spot where the first person's eye is and have the first person back up until the card is where their eye was with the arm still extended. Placing the meter stick on the wall with the first person holding one eye closed, note how many cm of the meter stick can not be seen by the first person. The first person will then back up one more arms length



and again record the amount of the ruler that cannot be seen. Repeat several more times. Record your results, determine the model, and complete the worksheet.

CARD LAB TWO



Materials: Each group will need a meter stick and a three by five card.

Instructions: Like Card Lab One distances will be measured in arms lengths. However this time the card will always be one arm's length from the wall (held by another person). The Observer (first person in Card Lab One) will vary the number of arms lengths their eye is **form the card** (not from the wall as in Card Lab One). Record the amount of the meter stick that can not be seen as in Card Lab One for distances of ½ arms length, one arms length, 2 arms length, 3 arms length as well as 4 and 5. Record your results, determine the model, and complete the worksheet.

HANDS ACROSS AMERICA

Materials: One stop watch.

Instructions: Four students will hold hands in a line. When the timer says go the person at the head of the line will squeeze the hand of the next person. When that person feels the squeeze of the first person, they will squeeze the hand of the next person in a chain reaction. When the last person feels their hand being squeezed, they will raise their empty hand and the timer will stop timing. Record the number of people and the time required. Add four more people and repeat. Continue until everyone is a part of the chain except the timer. (It is best if this experiment is done with the people in the chain have their eyes closed.) Record your results, determine the model, and complete the worksheet.

HEADS OR TAILS

Materials: Each group will need coin.

Instructions: Flip the coin 5 times and record the total number of times is landed heads. Flip the coin five more times and record the total number of heads for the ten tosses. Similarly record the number of heads after 15 tosses, 20 tosses, 25 tosses, 30 tosses, 35 tosses, 40 tosses, 45 tosses, and 50 tosses. Record your results, determine the model, and complete the worksheet.

THE TOSS OF A DIE

Materials: Each group will need a die.

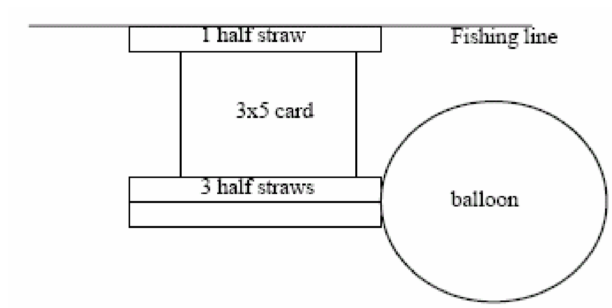
Instructions: Pick your favorite side of your die. Toss a die 5 times and record the total number of times your favorite side landed up. Toss the die five more times and record the total number of outcomes showing your favorite side for the ten tosses. Similarly record the number after 15 tosses, 20 tosses, 25 tosses, 30 tosses, 35 tosses, 40 tosses, 45 tosses, and 50 tosses. Record your results, determine the model, and complete the worksheet.

IT'S ALL KNOTTED UP

Materials: Each group will need a meter stick and a piece of rope about two meters long and ¼ to ½ inch in diameter.

Instructions: Tie one knot in the rope and record the length of the rope. Tie a second similar knot and now record the length. Continue for three, four, five, six, and seven knots. Record your results, determine the model, and complete the worksheet.

THE BALLOON SLED



Materials: Each group will need a meter stick, 3x5 card, a balloon, two plastic drink straws, some tape, and 10 meters of fishing line.

Instructions: Cut both straws in half. Tape three of them together. Slip the balloon opening onto one end of the three straws and tape the balloon tightly sealing the balloon to the straws. Tape the half straw to one side of the 3x5 card on the 5 inch side and tape the three straws with the balloon to the other five inch side of the card. Slip the fishing line through the single straw. Inflate the balloon to various diameters and record how far the balloon sled slides along the tightly held horizontal fishing line. Determine the model, and complete the worksheet.

CREATE YOUR OWN RELATIONSHIP

Materials: Each group's material list will vary with their choice of experiment they will do.

Instructions: Choose a situation that is related to your interests where you believe two variables are somehow dependent on each other. Make a prediction on which model will match their relationship. Perform the experiment. Record your results, determine the model, and complete the worksheet.

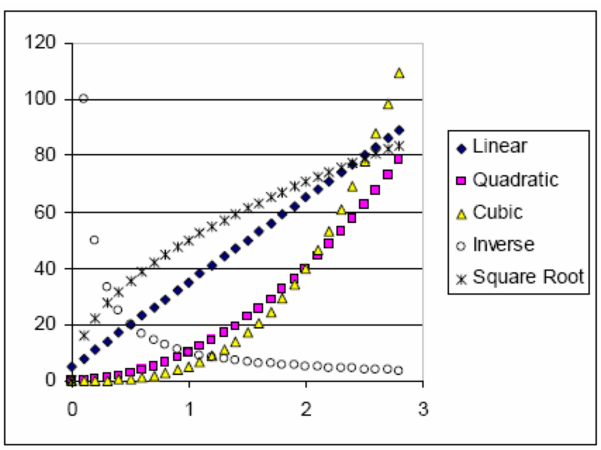
THE BALL BOUNCE

(An example of how I put my labs together. For the rest of them, email me at millerion@cleelum.com.)

Materials: Each group will need a meter sticks and a ball. A variety of balls that bounce well can be used. Suggested balls are: ping-pong balls, golf balls, tennis balls, volleyballs, whiffle balls, super balls, marbles, basketballs, etc.

Instructions:

1. Drop your ball from 10 cm, 20 cm, 30 cm… 100 cm. The drop height should be measured from the bottom of the ball to the surface the ball is being bounced off of. You may want to drop the ball from each height several times until your group agrees on the bounce height. Record your results in the upper left hand corner of your data sheet. Remember that x is the control variable, i.e. the variable that you are controlling and y is the dependant variable, the variable that is a result of the experiment with your choice of the control (independent) variable.
2. After you have collected your data, graph your data, making sure that you label your axes, showing the value of your tick marks and the units. Give your graph a title.
3. Determine which mathematical model best fits the data: linear, quadratic, cubic, inverse, or square root.



◊ If the data appears **linear**

1. draw your best fit line
2. find two representative points on your line preferably one near the beginning of the line and one near the end of the line
3. use the coordinates in the x, y table at the top of your work sheet to determine your linear model both in the slope intercept form and the general form, check whether it is reasonable to consider this a direct variation

◊ If the data appears **quadratic**, **cubic**, or **square root**

1. Fill out the table at the right of your graph if **quadratic** (lower right if cubic or square root). Use your original data, square your x values in the x^2 column (cube x if you think your data is **cubic**, square root of x if you think your data is the **square root** model).
2. On a separate graph, graph this data. If you have chosen the correct model, the graph should appear to be linear. Proceed by doing the linear model steps above.
3. Change your independent variable to correspond to the model you have chosen: change x to x^2 for the quadratic model, x to x^3 for the cubic, x to $\sqrt{x}$ for the square root model

◊ If the data appears **inverse**

1. Transfer the data to the inverse table
2. Multiply the control and dependent variables and enter the results in the x*y column
3. Your choice is correct if the values are relatively close to each other and your model will be x*y = the average vale of the x*y column

4. Answer the questions on the back of the lab worksheet. Be thoughtful in your answers. Remember that discrete values include only specific values and can not be values in between those where continuous can include all of the values in between. Domain is the reasonable values for the control variable. Sometimes common sense is used to determine the limits or extent of the domain. Common sense is also important to use when determining the range, the limits or extent of the dependent variable.
5. Compare your slope to others. What do you think the meaning of the slope is in this lab?