THE BIRTHDAY PARADOX

Here's a fun and easy application of probability to show the odds are good that two people in a relatively small group will share the same birthday.

Let's say you asked me when I celebrated my birthday, and I replied, "guess." If you were nice enough to play along, you would very probably guess wrong. Ignoring leap years, there are 365 days in a year, and I only celebrate my birthday on one of those days. The odds of you correctly guessing my birthday is 1 in 365 (or .003%).

That's an easy concept to grasp. It makes sense that you're not likely to guess my birthday.

Now let's say you know my birthday. What are the odds that the next person you meet on the street will share the same birthday as me? Again, the odds are abysmal: 1 out of 365. Therefore it seems very unlikely you'll find two people who share the same birthday, right? Well, not necessarily.

Let's say you know a group of 10 people. What are the odds that two of them share the same birthday? Without doing any math, it just seems the odds are low. How about 20 people? Or 30 people? Are the odds of two people sharing a birthday really low? How large does a group have to be until it actually becomes likely that two people actually DO share a birthday?

The answer may surprise you. But before we calculate it, let's predict something easier: dice.

You pick up two fair dice, give them a shake, and roll. What are the odds of rolling a match? (That's sort of like two people sharing a birthday.) One strategy is to first calculate the odds of NOT rolling a match and then subtract from 1. That provides the odds of the opposite event (ie, rolling a match). Think of flipping a coin. The odds of it landing heads are 1/2. Therefore the odds of it NOT landing heads are 1/2. Same principle applies here. Let's do it:

First calculate all the possible combinations of two rolled dice:

11 12 13 14 15 16

21 22 23 24 25 26

31 32 33 34 35 36

41 42 43 44 45 46

51 52 53 54 55 56

61 62 63 64 65 66

Count the combinations and you get 36. We could have achieved the same result by multiplying 6 x 6. So there are 36 possibilities. How many of those possibilities do NOT provide a match?

xx 12 13 14 15 16

21 xx 23 24 25 26

31 32 xx 34 35 36

41 42 43 xx 45 46

51 52 53 54 xx 56

61 62 63 64 65 xx

Count the possibilities and you get 30. Again we could have achieved the same result by multiplying 6 x 5. The first die has 6 ways it can land. The second die only has 5 ways to land if it is to satisfy the condition of NOT matching. Therefore there are 6 x 5 combinations of dice not matching.

So the odds of rolling two dice that do NOT match is 30/36, or 5/6. The odds of rolling the opposite situations -- two dice that DO match -- are consequently 1 - 5/6, or 1/6.

It would have been easier to just count the number of matches in our table, but the mathematical method will come in handy when calculating the odds of two people in a group having the same birthday. Speaking of which, let's do that now.

Pretend we have a slightly overcrowded classroom of 30 students. What are the odds of any two of the kids having the same birthday? As we did with the dice, let us first count the possible combinations, except minus the table. The total combinations of dice rolls was 6 x 6. Likewise, the total combination of birthdays is 365 x 365 x 365 x....(30 times), or more succinctly 36530. (Again we ignore leap years.) That's our denominator. Without calculating we can quickly see that's a huge number.

Now, as before, let's calculate the number of possibilities that do NOT provide a matching birthday. The first person states his birthday. For the next person to NOT have the same birthday, she has to pick from 364 days. The next person must choose from 363 days. And so on. Again were trying to calculate the number of combinations in which no one has a matching birthday. For the dice, the calculation was 6 x 5. Here we do the same thing:

365 x 364 x 363 x 362 x ... x 339 x 338 x 337 x 336

We can write this more concisely as follows:

365 x 364 x 363 .... (365 - n+1)

where n equals the number of kids in the classroom, in this case 30. Calculate and you have our numerator.

Put the two together to find the odds of NOT finding two kids with the same birthday in a group of 30 kids:

$$\frac{365 \times 364 \times 363 \ldots (365 - n + 1)}{365^{30}} = .292$$

There's about a 30% chance that you will NOT find two kids who share the same birthday in a group of 30. Therefore the opposite situation, that you WILL find two kids with the same birthday in a group of 30, is a whopping 70% (1 - .3 = .7). In fact, we find that in a group of 23 kids, your odds are better than 50% to find two people with the same birthday.

And that's the Birthday Paradox. It doesn't seem possible that the odds should be so good to find two people with the same birthday in such a relatively small group. But they are. Remember, these are not the odds of finding someone with the same birthday as YOU in a group of 30 people. These are the odds of finding ANY two people out of 30 who share a birthday.

Birthday Math Factoids

Neat math factoids about various birthdays:

your 3^{19} th second when you were 36y 10m,

your 2^{30} th second when you were 34y 9d,

your billionth second when you were 31y 8m,

your 10,000th day when you were 27y 4m,

your 3^{3} rd year when you were 27y 0m,

your e^{pi} th year when you were 23y 1m,

your 4^{4} th month when you were 21y 4m,

your 12! th second when you were 15y 2m,

your 7! th day when you were 13y 9m