Good N,E,W,S?

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Sometimes it is easier and more fun to take big steps than small ones.

Rather than introduce young children to ordinary algebra, maybe we can start them with a simplified linear algebra. And rather than introducing negative numbers on a line, why not introduce the notion of opposites in two dimensions – or more?

I have had enough success with what follows to be willing to suggest that others try it and test it. While I don't have evidence to support its immediate use in lots of classrooms, I see no reason to think that trying it could be harmful. The activities at least provide the chance to practice and use addition, subtraction, and multiplication of natural numbers. They don't require division or fractions.

The computer scientist and educator Seymour Papert developed the programming language LOGO for kids. He cited the Swiss psychologist Jean Piaget in suggesting that children may learn some abstract subjects, like programming, better if they can not only picture examples in that subject but can also picture themselves as being in these pictures and acting therein. In LOGO, he called the activity "thinking turtle." We will try to do something similar here with algebra. I don't think I would have come up with what follows had I not known a little about LOGO and had an even smaller acquaintance with Richard Pattis's robot Karel.

We can introduce vectors to children as kinds of movements, what physicists call displacements and mathematicians sometimes refer to as translations. The great textbook by Paul Halmos treats vectors this way, if I remember correctly from many decades ago.

We can let the capitol letter N stand for taking one step to the North, and similarly for E, S, and W. We can think of "and" as meaning "and then" and write it as a plus sign. Children can then use expressions like N+N+N+E+N meaningfully.

In my experience, no child has had any difficulty in using, say, 3N to mean and stand for N+N+N and then using expressions like 3N+4E+S+W.

In classrooms with square tiled floors, kids can carry out the "walk" that an expression like 3N+4E+S+W describes. They can draw walks on centimeter square graph paper. Multiple copies, if they want, maybe in different colors.

The walks 3N+E and E+3N are clearly different walks, but they end you up in the same place (provided you start in the same place). Kids can then understand why we might want to treat them as equivalent for some purposes and why we might write this fact as 3N+E = E+3N.

We can then ask kids if it is true or not that

3N+4E+S+W = 2N+2E.

We can ask them to find shorter walks that are "equal" to long ones.

We can tell kids that Katie's favorite dance step is 2E+N, like a waltz step or a knight's move in chess. They can write K = 2E+N and understand what 2K+3W means. We might ask them if they can find a walk X for which it is true that 3K + X = 5S. They might notice that if 5K + (something) = 2K + (something else), then 3K + (something) = (something else).

We can talk of the origin of a walk as being where you start and use capital O as the name of a walk in which you don't move at all, shamelessly exploiting the fact that it looks suspiciously like a zero.

Simplifying expressions like 3N+4E+S+W leads to the idea that N and S are opposites, as are E and W. We can write N+S = O and E+W = O. I have had middle school students consider yellow submarines that can also go Up and Down, and Star Trek space ships that can go Forwards and Backwards in time. Four dimensions!

We can introduce the idea of the opposite of a walk and use a minus sign to designate it, so that –N = S and –J = 2W+S.

After kids simplify expressions for a while, they might find that some general rules are useful and always true. Rules like

A+B = B+A, -(-(A)) = A, and n(mA+kB) = nmA+nkB.

They might try various ways of convincing each other that the rules hold. They might be able to come up with counterexamples to the idea that n(A+B) = nA+B.

If desired, this way of doing things makes it natural to introduce "arrow adding" and parallelograms to middle school students.

If we replace all of the capital letters we have used with lower case ones and think of them as standing for numbers, retracing what we have done amounts to a fair amount of algebra, don't you think? Don't you agree that N,E,W,S might be good in the classroom?